Elastohydrodynamic Lubrication Analysis of Journal Bearings Using CAD

Toshihiro OZASA[*1], Akira KINOSHITA[*2], Yuji HIROSE[*3] and Masatoshi NIIZEKI[*4]

*1 Department of Mechanical Engineering, Faculty of Engineering,
Osaka Electro-Communication University
18-8 Hatsu-cho, Neyagawa-shi, Osaka 572-8530, JAPAN
ozasa@isc.osakac.ac.jp
*2 Graduate School of Osaka Electro-Communication University, Now, PSG., Inc.
*3 Graduate School of Osaka Electro-Communication University
18-8 Hatsu-cho, Neyagawa-shi, Osaka 572-8530, JAPAN
*4 Department of Electro-Mechanical Engineering, Faculty of Engineering,
Osaka Electro-Communication University
18-8 Hatsu-cho, Neyagawa-shi, Osaka 572-8530, JAPAN
niizeki@isc.osakac.ac.jp

Abstract

In the calculation of the Elastohydrodynamic Lubrication, EHL, of a journal bearing, a compliance matrix which expresses the relation between deformation and oil film pressure on a bearing surface is necessary. However, the derivation and preparation of the compliance matrix is a difficult task. In this study, a method of deriving the compliance matrix using the structural analysis in a Three Dimensional Computer Aided Design software, 3D-CAD, is described and performed using CATIA V5. The EHL analysis of the con-rod bearing using the compliance matrix is performed under the dynamic load of an engine and the deformations and oil film pressure distributions on a bearing surface are obtained over the engine cycle. The stress distributions in the con-rod are studied using the structural analysis in the 3D-CAD. In this case, the pressure distributions on the bearing surface are used to calculate the boundary condition, namely the nodal force distributions on the bearing surface. Finally, the change of the maximum stress in the con-rod under the engine operation is shown for the design of the con-rod.

Keywords: elastohydrodynamic lubrication, journal bearing, computer aided design, structural analysis, compliance matrix, stress analysis, con-rod

1 Introduction

Deformation of bearings used under high loading conditions like engines could not be neglected. Therefore the Elastohydrodynamic Lubrication, EHL, analysis of a journal bearing is necessary and the study of EHL has been started [1]. In the calculation of the EHL, a compliance matrix which expresses the relation between deformation and oil film pressure on a bearing surface is derived from a structural analysis and improves calculation efficiency [2]-[6]. However, the derivation and preparation of the compliance matrix take a lot of task. This is an obstacle to apply the EHL to design of a machine with dimensional changes.

In these days, the design of a machine is performed using a Computer Aided Design, CAD, and a recent CAD includes the function of a structural analysis. Therefore in this study, a method of deriving the compliance matrix using the structural analysis in a Three Dimensional Computer Aided Design software, 3D-CAD, is described and performed using CATIA V5.

The EHL analysis of the con-rod bearing using the compliance matrix is performed under the dynamic load of an engine and the deformations and oil film pressure distributions on a bearing surface are obtained over an engine cycle. The stress distributions in the con-rod are studied using the structural analysis in the 3D-CAD. In this case, the pressure distributions on the bearing surface are used to calculate the boundary condition, namely the nodal force distributions on the bearing surface. Finally, the change of the maximum stress in the con-rod under the engine operation is shown for the design of the con-rod.

2 EHL of journal bearings

In a lubrication analysis, Reynolds equation is used [2]-[5], [7].

$$\frac{1}{R}\frac{\partial}{\partial\theta}\left\{\frac{h^3}{12\mu R}\frac{\partial p}{\partial\theta}\right\}+\frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right\}=\frac{1}{R}\frac{\partial}{\partial\theta}\left(\frac{hU}{2}\right)+\frac{\partial h}{\partial t} \quad (1)$$

where h is clearance [m], p is pressure [Pa], R is radius [m], t is time [s], U is velocity on a journal surface [m/s], θ is bearing angle [rad], and μ is viscosity [Pa s]. Share stress τ [Pa] is

$$\tau=\mu\frac{\partial u}{\partial y} \quad at \quad y=h \quad (2)$$

where u [m/s] is velocity of oil at y [m] in oil film thickness.

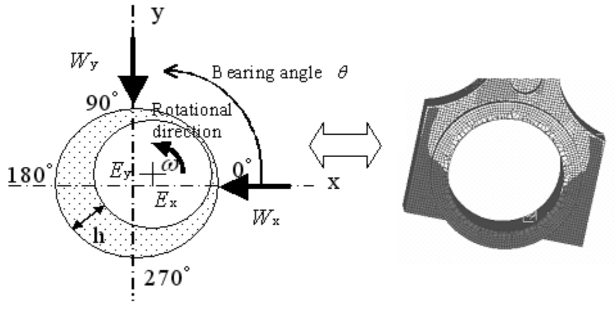


Fig. 1 Journal bearing and EHL

Figure 1 shows a journal bearing with explanation of the EHL. W_x and W_y are loads on big end bearing in x and y directions, respectively [N].

In the case of a con-rod, moment of force on a small end by means of pressure, share stress and W_x in big end bearing is zero. Therefore they do not any influence on load in the cylinder direction of an engine. The force balances on a big end bearing are

$$\begin{aligned} W_x &= \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_0^{2\pi}\left\{p\cos\theta-\tau\left(\sin\theta-\frac{R}{l}\right)\right\}Rd\theta dz\ , \\ W_y &= \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_0^{2\pi}\left\{p\sin\theta+\tau\left(\cos\theta-\frac{R}{l}\tan\alpha\right)\right\}Rd\theta dz \end{aligned} \tag{3}$$

where b is bearing width [m], l is length between centers of the big and small ends [m], α is the angle between cylinder and con-rod axes.

Oil film thickness between a journal and a bearing is described as

$$\begin{aligned} h &= R-\sqrt{r^2-\left(E_x\sin\theta-E_y\cos\theta\right)^2} \\ &\quad -E_x\cos\theta-E_y\sin\theta+L \end{aligned} \tag{4}$$

where r is radius of the journal [m] , E_x and E_y are eccentricities of a journal center in x and y directions [m] and L is deviation from a clearance circle [m]. L can be treated as static deformation and is described with a matrix and vectors.

$$[\boldsymbol{L}]=[\boldsymbol{C}][\boldsymbol{p}]+[\boldsymbol{L_0}] \tag{5}$$

where $[\boldsymbol{L}]$ is the deviation vector from a bearing circle, $[\boldsymbol{C}]$ is compliance matrix which expresses the relation between elastic deformation on a bearing surface and oil film pressure distribution vector $[\boldsymbol{p}]$ and $[\boldsymbol{L_0}]$ is the shape of bearing expressed by deviation from the clearance circle. The $[\boldsymbol{C}]$ is derived from structural analysis.

Equations (1) and (3) are numerically solved for the region of p>0 with the oil film thickness calculated from Eqs. (4) and (5). Pressure in the cavitation is assumed to be zero, p = 0. **Figure 2** shows calculation flow of the EHL. In the EHL analysis, the oil film thickness changes with elastic deformation. Therefore calculation must be continued till convergence in elastic deformation and the EHL results are obtained.

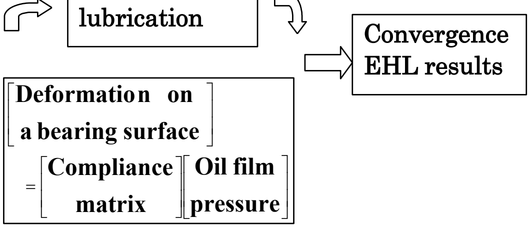


Fig. 2 Calculation flow of EHL using compliance matrix

3 Compliance matrix

In many cases, a structure is analyzed with FEM. The relation between forces and deformations in the structure is described by liner relation using vectors and matrixes. $[F]$ is a force vector on all nodes, $[\delta]$ is a deformation vector in all nodes in a structure and $[K]$ is a stiffness matrix. The relation between the forces and deformations of all nodes in the structure is

$$[F]=[K][\delta]\ . \tag{6}$$

When $[\boldsymbol{D}]$ is defined as $[\boldsymbol{K}]^{-1}\equiv[\boldsymbol{D}]$,

$$[\delta]=[\boldsymbol{D}][\boldsymbol{F}]\ . \tag{7}$$

To make a compliance matrix, a unit force is applied on bearing surface [2], [3]. $[\boldsymbol{D'}]$ is the matrix which express the relation between deformation of structure and force on load points of a bearing surface in x and y coordinates and $[\boldsymbol{F_b}]$ is the force vector of node on a bearing surface in x and y directions. Deformation is expressed with

$$[\boldsymbol{\delta}]=[\boldsymbol{D}'][\boldsymbol{F_b}]\ . \tag{8}$$

When a unit force is loaded in x direction at a node on a bearing surface, deformation is a column of the $[\boldsymbol{D}']$ corresponds to the unit load. When a unit force is loaded in y direction at a node on the bearing surface, deformation is a column of the $[\boldsymbol{D}']$ corresponds to the unit load.

The nodal points are determined by auto-mesh and change with every calculation, so that they are different from the calculation mesh of the lubrication analysis. Therefore the deformation must accord to the calculation mesh of the lubrication by means of interpolations.

$[\boldsymbol{\delta_{xyL}}]$ is the deformation vector on a bearing surface in x and y directions according to the calculation mesh of lubrication derived with the interpolations of the $[\boldsymbol{\delta_b}]$ which is the deformation vector on a bearing surface selected from the deformation vector $[\boldsymbol{\delta}]$, $[\boldsymbol{D''}]$ is the matrix which expresses the relation between deformation on the mesh points of lubrication and force on load points of bearing surface in x and y coordinates. Deformation of lubrication mesh in x and y directions is expressed with

$$[\boldsymbol{\delta_{xyL}}]=[\boldsymbol{D''}][\boldsymbol{F_b}]\ . \tag{9}$$

When $[\boldsymbol{\Delta A}]$ is the matrix which express the area of lord points on bearing surface, $[\boldsymbol{T_1}]$ is the matrix which transfer radial force to the forces in x and y directions, $[\boldsymbol{T_2}]$ is the transfer matrix which expresses the relation between lord points and lubrication mesh on bearing surface, $[\boldsymbol{F_r}]$ is a radial force vector on bearing surface and $[\boldsymbol{E}]$ is unit matrix, the following equations are obtained.

$$[\boldsymbol{F_r}]=[\boldsymbol{T_2}][\boldsymbol{\Delta A}][\boldsymbol{p}] \tag{10}$$

$$\begin{aligned} [\boldsymbol{\delta_{xyL}}] &= [\boldsymbol{D''}][\boldsymbol{F_b}]=[\boldsymbol{D''}][\boldsymbol{T_1}][\boldsymbol{F_r}] \\ &= [\boldsymbol{D''}][\boldsymbol{T_1}][\boldsymbol{T_2}][\boldsymbol{\Delta A}][\boldsymbol{p}] \end{aligned} \tag{11}$$

In the above equations, load points should be taken as mesh points of lubrication, namely $[T_2]=[E]$.

When $[\delta_{av}]$ is averaged mean vector of deformation and $[T_{av}]$ is the transfer matrix of averaged mean of deformation on bearing surface in x and y directions, relative deformation of a bearing is

$$\lfloor \delta_{xyL} \rfloor - [\delta_{av}] = ([E]-[T_{av}]) \lfloor \delta_{xyL} \rfloor = ([E]-[T_{av}])[D''][T_1][T_2][\Delta A][p] \quad . (12)$$

When $[T_r]$ is the transfer matrix of deformation on bearing surface from x and y directions to radial direction, the compliance is expressed with

$$[C]=[T_r]([E]-[T_{av}])[D''][T_1][T_2][\Delta A]. \quad (13)$$

When $[\delta_r]$ is deformation in radial direction, the deviation vector from a bearing circle is

$$[L]=[\delta_r]+[L_0]=[C][p]+[L_0] \quad . \quad (14)$$

This is Eq. (5). **Figure 3** shows the flow chart of making the compliance using the 3D-CAD.

In the practice, the structure is con-rod. Inside of the small end is fixed and the deformations of the surface of big end bearing are calculated in the 3D-CAD when 1 [N] is applied every node on the bearing surface each in x or y direction. **Figure 4** shows the dimension of the con-rod. **Figure 5** (a) shows the structure model of the 3D-CAD. **Figure 5** (b) shows the auto mesh model for the structural analysis in which element size is about 1 [mm].

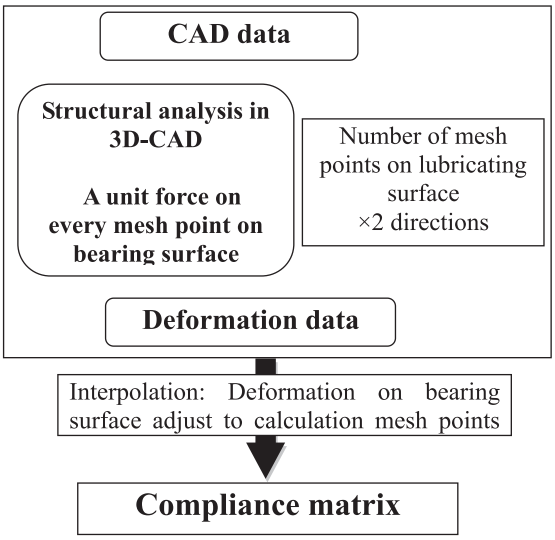


Fig. 3 Flow of making a deformation-pressure matrix, compliance matrix, using structural analysis in Three Dimensional Computer Aided Design Software

The structure analysis was done with the mesh on the bearing surface which is 72 in circumferential direction and 11 in axial direction. The number of times of the structure analysis is the number of the mesh points in the lubrication analysis times 2 in directions. In this case, it is 1584. Con-rod is symmetrical shape in left and right and in front and behind about the rod axis. The product of 37 in circumference, 6 in width and 2 in directions is the number of times of the structure analysis which is reduced to be 444.

4 Results

The EHL analysis of the con-rod bearing using the compliance matrix is performed under the dynamic load of an engine and the deformations and oil film pressure distributions on a bearing surface are obtained over the engine cycle. In the calculations, suction top dead center is defined as crank angle of 0 degrees after top dead center, 0 [°ATDC]. The stress distributions in the con-rod are studied using the structural analysis in the 3D-CAD, CATIA V5, using the nodal forces on the bearing surface as the boundary condition. The nodal force distributions on the bearing surface are derived from the pressure distributions obtained from the EHL analysis.

Figures 6 to 9 show the results of engine speeds 1000 [rpm] to 4000 [rpm] at 390 [°ATDC] when the maximum stress appears by means of the explosion in the cylinder. In these figures, the deformation on the bearing surface is shown in (a), oil film pressure is shown in (b) and stress distributions in the con-rod is shown in (c). The loads on the con-rod bearing are shown in **Table 1**.

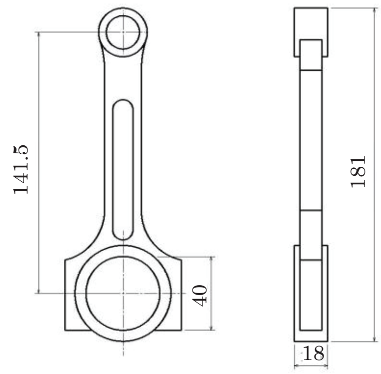
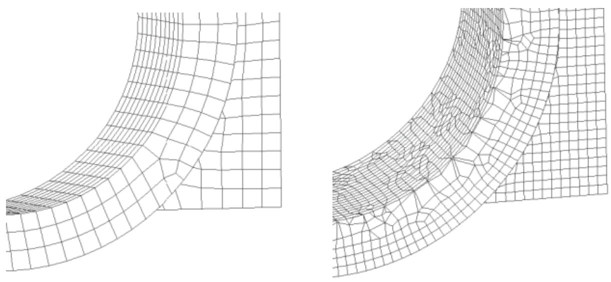


Fig. 4 Dimensions of con-rod [mm]

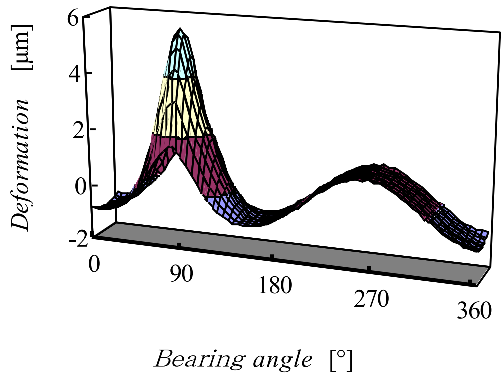


(a) 3D-CAD data **(b) FEM structural model**

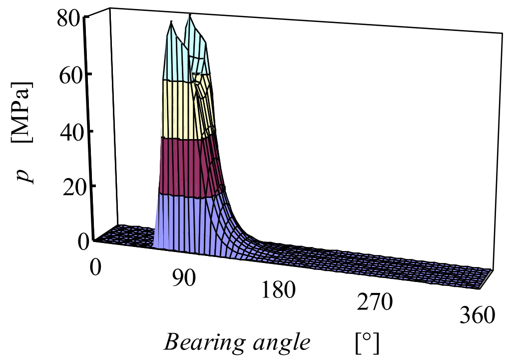
Fig. 5 Mesh on bearing surface

Table 1 Load on con-rod bearing at 390 [°ATDC]

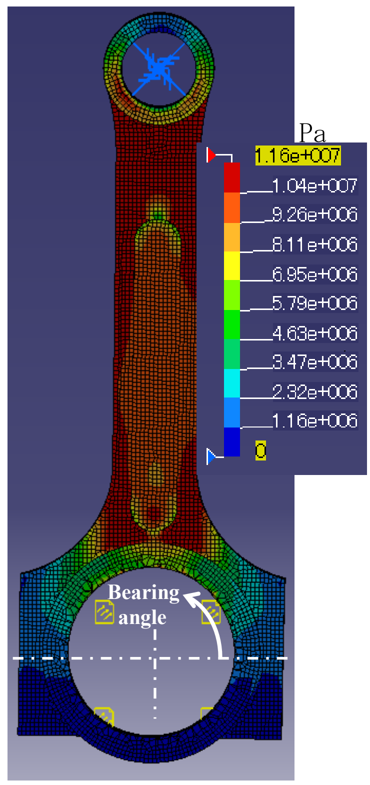
	Load	
Engine speed [rpm]	W_x [kN]	W_y [kN]
1000	-0.05791	15.94
2000	-0.2329	14.88
3000	-0.5245	13.1
4000	-0.9329	10.62



(a) Deformation on bearing surface

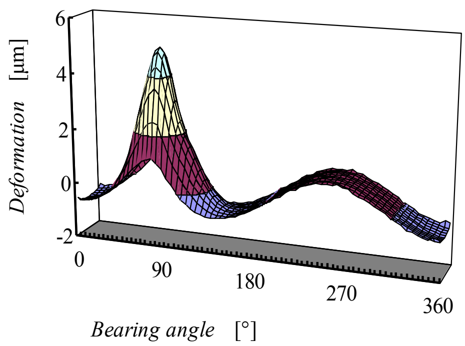


(b) Oil film pressure

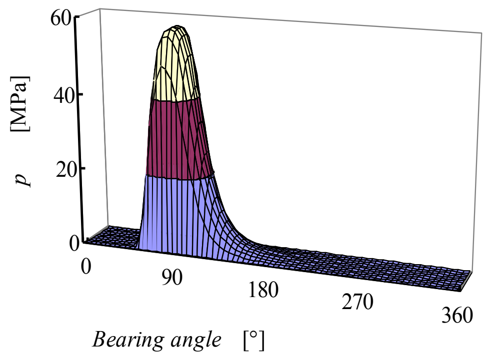


(c) Stress distribution of con-rod

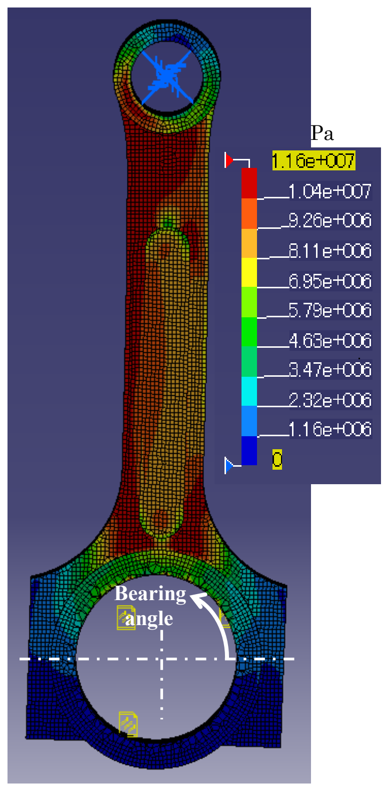
Fig. 6 EHL analysis and structural analysis at 390°ATDC under engine speed of 1000rpm



(a) Deformation on bearing surface

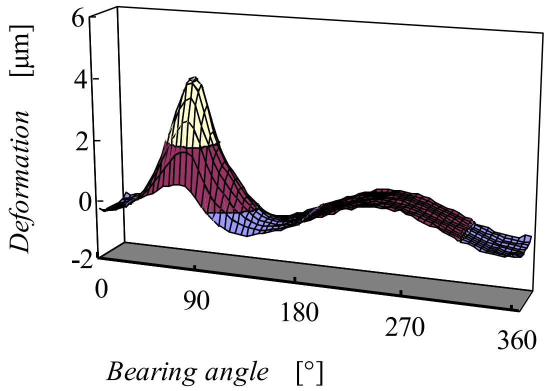


(b) Oil film pressure

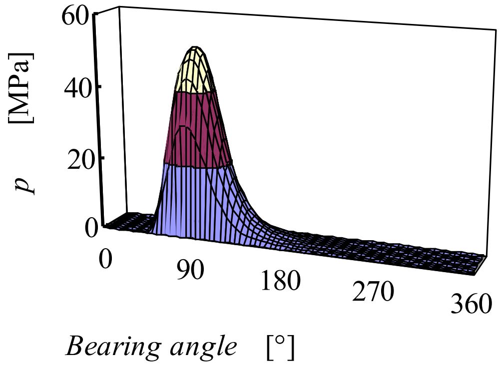


(c) Stress distribution of con-rod

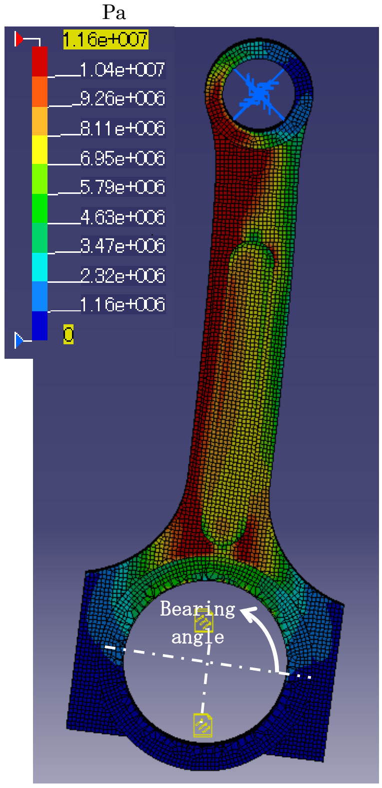
Fig. 7 EHL analysis and structural analysis at 390°ATDC under engine speed of 2000rpm



(a) Deformation on bearing surface

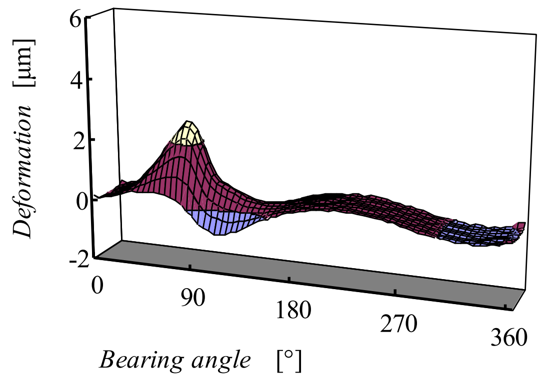


(b) Oil film pressure

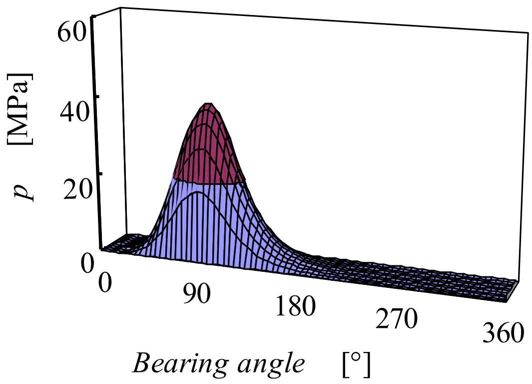


(c) Stress distribution of con-rod

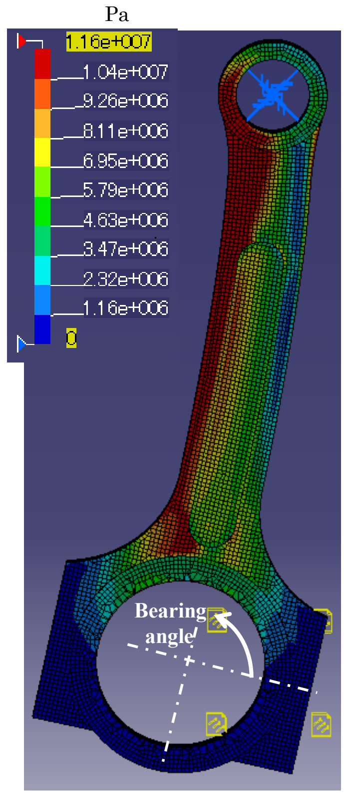
Fig. 8 EHL analysis and structural analysis at 390°ATDC under engine speed of 3000rpm



(a) Deformation on bearing surface

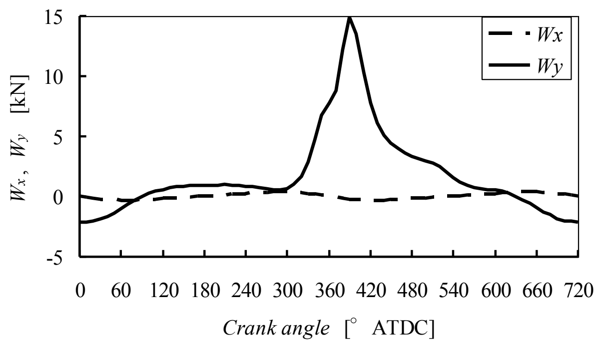


(b) Oil film pressure

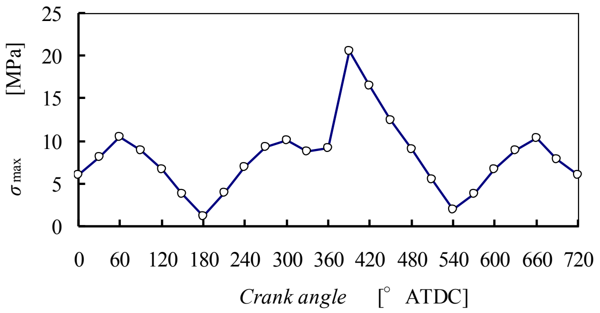


(c) Stress distribution of con-rod

Fig. 9 EHL analysis and structural analysis at 390°ATDC under engine speed of 4000rpm



(a) Bearing load



(b) Maximum stress in con-rod

Fig. 10 Bearing load and maximum stress in con-rod under engine speed of 2000 rpm

Figure 6 is the result at 1000 [rpm]. As shown in (a), the load in the direction of con-rod axis, W_y, is high and maximum deformation is about 6 [μm]. As shown in (b), the pressure is high in the direction of load and the peak pressures appear at both ends in the bearing width. This is a peculiar case in the EHL like as reported in the past work [7]. The oil film thickness in the both ends is quite low. As shown in (c), the stress distribution in the con-rod is quite high all over the rod. **Figure 7** is the result at 2000 [rpm]. As shown in (b), the peak pressure appears at middle in the bearing width. As shown in (c), the stress distribution in the con-rod is like as that at 1000 [rpm] but slightly lower. **Figure 8** is the result at 3000 [rpm]. As shown in (b), the peak pressure appears at middle in the bearing width. As shown in (c), the stress is slightly high in the left side of the rod by means of bending. **Figure 9** is the result at 4000 [rpm]. As shown in (c), the bending of the rod is large and the stress is quite high in the left side of rod. The relation between the EHL analysis and the stress analysis is clarified and importance of the relation could be understood. Finally, the change of the maximum stress in the con-rod under the engine cycle at the 2000 [rpm] is shown in **Fig. 10** for the design of the con-rod. The engine is frequently operated near this speed in a usage. In this speed, the maximum stress in the con-rod is highest near 390 [° ATDC] because of the combustion in the cylinder. The highest maximum stress is lower enough in comparison with the property of steel materials.

5 Conclusions

The compliance matrix, which expresses the relation between elastic deformation on bearing surface and oil film pressure, is useful in the EHL analysis. The method of deriving the compliance matrix using the structural analysis in the 3D-CAD is described and is performed using CATIA V5. The EHL analysis of the con-rod bearing using the compliance matrix is performed under the dynamic load of the engine. The deformations and oil film pressure distributions on a bearing surface are obtained over the engine cycle.

The stress distributions in the con-rod are studied using the structural analysis in the 3D-CAD. In this case, the pressure distributions on the bearing surface are used to calculate the boundary condition, namely the nodal force distributions on the bearing surface. Finally, the change of the maximum stress in the con-rod under the engine operation is shown for the design information.

This method is confirmed to be useful. This is because the stress analysis can be done with the practical boundary conditions.

References

[1] Fantino, B., Godet, M. and Frene, J., "Dynamic Behavior of an Elastic Connecting-Rod Bearing – Theoretical Study", SAE Paper 830307, (1983).

[2] Goenka, P. K. and Oh, K. P., "An Optimum Short Bearing Theory for the Elastohydrodynamic Solution of Journal Bearings", Transactions of the ASME, Journal of Tribology, Vol. 108 (1986), pp. 294-299.

[3] Mclvor, J. D. C. and Fenner, D. N., "Finite Element Analysis of Dynamically Loaded Flexible Journal Bearings: First Newton-Raphson Method", Transactions of the ASME, Journal of Tribology, Vol. 111 (1989), pp. 597-604.

[4] Ozasa, T., Yamamoto, M, Noda, T. , Mizutani, Y. , Yamada, M and Torii, H., "Elastohydrodynamic Lubrication Analysis of Con-Rod Big-End Bearings-Experimental Verification by Engine Test-", in Japanese, Proceedings of the Japan Society of Mechanical Engineers, No. 930-63, E, 2006(1993), pp.25-27.

[5] Ozasa, T., Noda, T. and Konomi, T. "Elastohydrodynamic Lubrication Model of Connecting Rod Big-End Bearing : Application to Real Engine"s, Transaction of the ASME journal of Tribology, Vol. 119, (1997), pp.568-578.

[6] Ozasa, T., "Mass Conserving Elastohydrodynamic Lubrication for Con-Rod Bearings with an Oil Passage", in Japanese, Transactions of the Japan Society of Mechanical Engineers, Vol. 68, 666, C(2002), pp. 613-620.

[7] Ozasa, T. , Suzuki, S., Noda, T. and Konomi, T. "Elastohydrodynamic Lubrication of Con-Rod Big-End Bearings: Comparison between Theory and Engine Test", Proceedings of the International Tribology Conference Yokohama 1995 (1996), pp. 1067-1072.

Received on December 31, 2013
Accepted on February 27, 2014