The 4th International Conference on Design Engineering and Science, ICDES 2017
Aachen, Germany, September 17-19, 2017

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Soft Landing Condition for Stair-climbing Robot with Hopping Mechanism
(Feasibility Discussion for Multiple Soft Landing Points)

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Abstract

We have proposed a fast stair-climbing robot with a simple hopping mechanism that uses vibration generated by a two-degrees-of-freedom (2-DOF) system. The robot, which consists of two (upper and lower) bodies connected by springs and wire, travels quickly using wheels mounted on its lower body and hops to climb stairs by releasing energy stored in the springs. The trajectories of the bodies during hopping depend on mechanical design parameters such as the reduced mass of the two bodies, the mass ratio, and the spring constant, as well as control parameters such as the initial spring contraction and horizontal velocity. This mechanism allows the robot to climb stairs quickly and economically and to land softly without the need for complicated controls. In our research to date, we have analyzed a defined soft landing point using equations of motion, clarified conditions, and showed the existence, that the soft landing point is created by a damping term such as friction during spring contraction phase. In this paper, in order to achieve the soft landing against multiple riser heights practically, we discuss our analysis of how many soft landing points are created using design/control parameters. Here, not only just one soft landing point but also two or three soft landing points were created mathematically and additionally, as an example, the practical design solutions were shown.

Keywords: stair-climbing, hopping mechanism, 2-DOF vibration system, soft-landing points

1 Introduction

Since surmountability, defined as the ability to surmount and overcome obstacles, is very important for robots traveling on rough terrain, various mechanisms such as a crawler- [1]-[2], legged- [3], hybrid- [4], and modular-type [5] machines have been proposed and developed. However, if we focus on a standardized or regular environment, such as common stairs in office buildings, we believe that a small, simple, agile, and specialized (rather than generalized) robot would be more suitable. Furthermore, we are convinced that such robots are not only theoretically possible but also can be developed practically.

Based on this viewpoint, we have limited the rough terrain to standardized stairs in order to develop a mobile observation system for office buildings, and have designed and developed a fast stair-climbing wheel-type robot with a hopping mechanism by a two-degrees-of-freedom (2-DOF) vibration system [6], [7]. Although slow stair-climbing robots larger than the riser of stairs have been developed previously, our small robot, which is almost the same height as the riser, has practically demonstrated a fast stair-climbing velocity of 1.2m/s using simple controls and passive soft-landing canceling the impact acceleration by the vibration. Meanwhile, we have also theoretically analyzed the existence of the soft landing point which makes the robot to land softly [8]. Note that a soft landing point is defined as one where the vertical velocity is 0m/s, the vertical acceleration is 0m/s^2, and the zero or negative vertical jerk (<0m/s^3) occurs at a desired height. We have clarified the soft landing condition, i.e., the relationship between the characteristics of existence and the design/control parameters of the robot, based on equations of motion. However, while this result shows the condition that the soft landing point exists, but it dose not clarify how many soft landing points are created under such condition. This is significantly important because if just one soft landing point exists under a certain condition, the stair-climbing robot can only land softly against one riser height and cannot land softly against other riser heights. On the other hand, if changing the control parameters can create multiple soft landing points, the stair climbing robot can land softly when encounting multiple different riser heights. That is because the control parameters such as the initial spring contraction can always be changed during locomotion, while the design parameters such as the mass and spring constant cannot be changed after fabrication.

In this paper, we report on our analysis of a stair-climbing robot that can land softly when surmounting stairs with multiple riser heights simply by changing the control parameters. Additionally, we clarify the creatable number of soft landing points based on conditions obtained via equations of motion and discuss the feasibility of finding soft landing points from the range of practical design/control parameters.

2 Proposed stair-climbing robot

2.1 Star-climbing robot overview

Figure 1 shows an example of the developed stair-climbing robot, which consists of upper and lower bodies (Bodies 1 and 2) connected by four shafts, four springs, and a wire [8]. The upper body has a wire reeling mechanism and electric devices such as a circuit and battery. The lower body has four wheels, two motors, and an acceleration sensor. The robot, which is 0.125m in width, 0.11m in length, and 0.33m in height, travels horizontally using its wheels, hops vertically to the next riser height by releasing the energy stored in the springs, and lands softly by canceling the descending velocity of the lower body using the ascending velocity of the vibration.

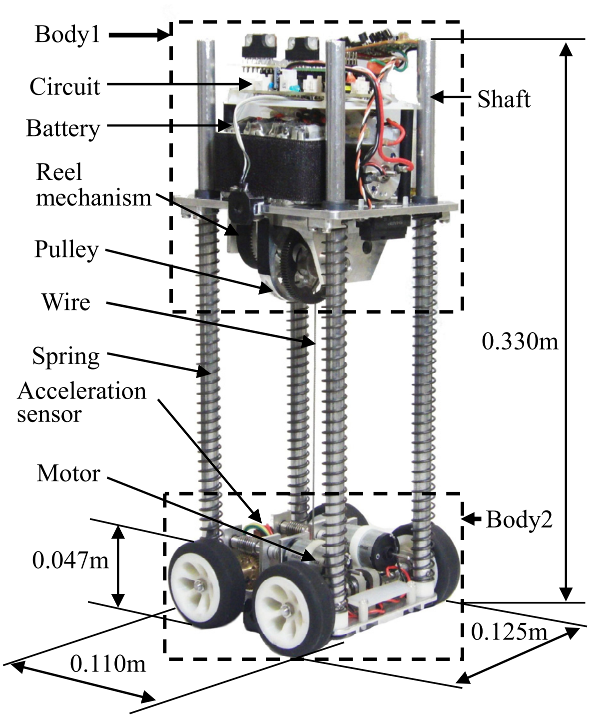

Fig. 1 Hopping robot overview [8]

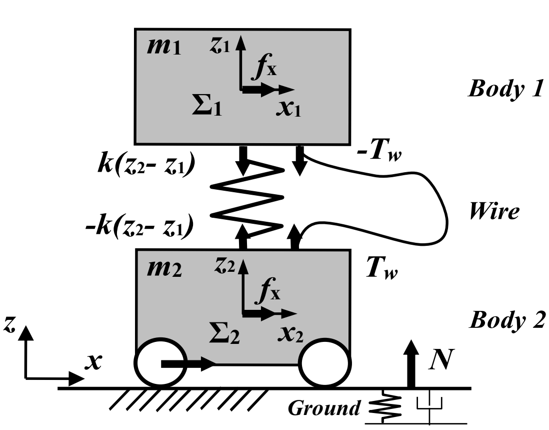

Fig. 2 Two dimensional model of hopping robot [8]

2.2 Two dimensional mathematical model

Figure 2 shows a simplified two-dimensional (2D) mathematical model of the stair-climbing robot shown in **Fig. 1**. The equations of motion is described as a 2-DOF spring mass system in the sagittal (x-z) plane

$$\begin{aligned} &(m_1+m_2)\ddot{x}_1=(m_1+m_2)\ddot{x}_2=f_x \\ &m_1\ddot{z}_1=k(z_2-z_1)-m_1 g-T_w-\mu_t F_f \\ &m_2\ddot{z}_2=k(z_1-z_2)-m_2 g+T_w+\mu_t F_f+N \end{aligned} \quad , \tag{1}$$

where (x_i, z_i) is a coordinate system for the i^{th} body, m_i is the mass, k is a spring constant, f_x is the motor force for horizontal travel, $\mu_t F_f$ is the friction of the shaft, N is the reaction of the ground, g is a gravitation constant (9.8m/s^2), and T_w is the wire tension. Note that the orientation of the robot (which is x_1=x_2) is neglected in this study, since the horizontal velocity is almost constant and the moment around the center of gravity is not generated [6]. Here, z_1–z_2=0 means the natural length of the spring.

2.3 Definition of soft landing

In this study, a soft landing is defined as a special landing in which the vertical velocity of the lower body is zero ($\dot{z}_2=0$), the acceleration is zero ($\ddot{z}_2=0$), and the jerk is zero or negative ($\dddot{z}_2\le 0$) at the landing height (z_2). Mathematically, this is the inflection and stationary point.

In [8], in cases where the wire tension is not controlled (T_w=0), the soft landing points exist during the spring contraction phase ($\dot{z}_1-\dot{z}_2$<0)

$$\pi/2<\omega\hat{t}+\phi-2n\pi<3\pi/2 \quad , \tag{2}$$

if and only if the shaft friction as Coulomb friction is defined as

$$\mu_t F_f=\begin{cases} \sigma & if \ \ 0<\dot{z}_1-\dot{z}_2 \\ 0 & if \ \ \dot{z}_1-\dot{z}_2=0 \\ -\sigma & if \ \ \dot{z}_1-\dot{z}_2<0 \end{cases} . \tag{3}$$

From this, the soft landing point is solved from eq. (2) and three conditions

$$\frac{M\sigma}{m_2}[\frac{h(4n+2)}{m_2 g}-\frac{(4n+2)^2\sigma}{2m_2 gk}-\frac{2}{k}]=H \tag{4}$$

$$\hat{t}=\frac{M\omega}{m_2 g}[h-\frac{(4n+2)\sigma}{k}]\sqrt{1-(\frac{m_2 g}{kh-(4n+2)\sigma})^2}+T \tag{5}$$

$$\frac{k}{m_2}[h-\frac{(4n+2)\sigma}{k}]\sin(\omega\hat{t}+\phi)-g=0 \quad , \tag{6}$$

where ω is the angular frequency, M is the reduced mass, and the constants, T and ϕ, determined via the initial conditions are

$$\omega = \sqrt{k/M}, \quad M = m_1 m_2 / (m_1 + m_2)$$
$$\sin\phi = \frac{m_2 g}{kh}, \ T = \frac{Mh}{m_2 g}\omega\sqrt{1-(\frac{m_2 g}{kh})^2} \quad . \tag{7}$$

Here, $\hat{t}$ is the landing time (t=0 is defined as the takeoff time), n is the non-negative integer denoting the iteration number of vibration, σ is the friction, h is the amplitude constant after takeoff (not the amplitude), and H is the landing height (the height of the stair riser).

Figure 3 shows an example of soft landing using the design/control parameters of k=1000, h=0.13, n=2, m_1=1.5351, and m_2=1.3761 obtained from above conditions. The thin and bold lines are the trajectories of Bodies 1 and 2, respectively, and the arrows show the direction of the spring force. The horizontal axis refers to the time. The traveling distance (x position) is calculated from the time multiplied by the constant horizontal velocity. Theoretically, this system achieves almost 0G impact acceleration landing at the height of H=0.2 ("A") during the spring contraction phase. Note that the shaft friction, σ, was set to 5.0N for the pilot experiment.

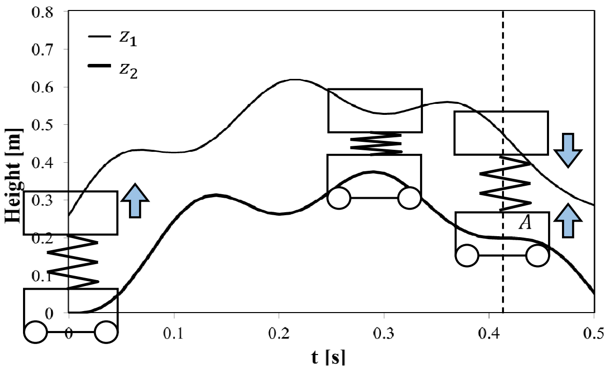

Fig.3 Trajectories of Bodies 1 and 2 during hopping

3 How many soft landing points are created

In [8], the existence of soft-landing point meeting eqs. (2), (4), (5), and (6) has been shown. In this paper, we clarify how many soft landing points are created mathematically and discuss their practical feasibility. For example, the creation of two different soft landing points means that the robot can land softly on both treads at two different riser heights.

As constraints, three design parameters: the upper and lower body masses, m_1, m_2, and the spring constant, k, cannot be changed after robot fabrication. Thus, the same design parameters must be used for all soft landing points. On the other hand, three control parameters: the amplitude constant after takeoff, h, the landing time, $\hat{t}$, and the iteration number of vibration, n, can be used independently for each riser height, H. Note that n is the non-negative integer, i.e, the discrete value. Meanshile, in a certain range (eq. (2)), three soft landing conditions (eqs. (4), (5), and (6)) exist for each riser height. Since the number of unknown variables (design/control parameters) is 3+3X and the condition number is 3X for the number of different riser heights, X, the number of unknown variables is always more than the condition number (3+3X>3X). Thus, infinite sets of design/control parameters for one soft landing point must exist mathematically, and infinite soft landing points must be creatable. However, since the iteration number of vibration, n, is discrete, the obtained n is hardly ever a non-negative integer. Hence, since n cannot be used as the arbitrary variable, the number of control parameters is realistically restricted to two. Therefore, since unknown variables are 3+2X and the condition number is 3X, the solution set must exist if and only if X<=3 (3+2X>=3X). Here, the soft landing conditions (eqs. (4), (5), and (6)) hold for only the spring contraction phase ($\dot{z}_1 - \dot{z}_2 < 0$) in the range of eq. (2) with the discrete variable, n, and are not a continuous function. From these results, we discuss the cases of X=2, 3, that is, the stair-climbing for two and three riser heights. Note that the case of X=1 have been already shown both mathematically and practically in [8].

4 Two kinds of soft landing points (X=2)

Let two different riser heights be H_a and H_b ($H_a \neq H_b$) and let the amplitude constants after takeoff and the landing times be h_a, h_b, t_a, and t_b, six conditions are obtained from eq. (4), (5), and (6):

$$\begin{aligned}
h_i &= \frac{m_2 g}{N_i}[\frac{m_2 H_i}{M\sigma} + \frac{N_i^2\sigma}{2m_2 gk} + \frac{2}{k}] \\
t_i &= \frac{M\omega}{m_2 gk}[kh_i - N_i\sigma]\sqrt{1-(\frac{m_2 g}{kh_i - N_i\sigma})^2} + T_i \\
E_i &= \sin(\omega t_i + \phi_i) - \frac{m_2 g}{kh_i - N_i\sigma} \\
&\sin\phi_i = \frac{m_2 g}{kh_i}, \ T_i = \frac{Mh_i}{m_2 g}\omega\sqrt{1-(\frac{m_2 g}{kh_i})^2} \\
N_i &= 4n_i + 2
\end{aligned} \tag{8}$$

where i=a, b, and E_i is the error function. The variables, m_1, m_2, k, h_a, h_b, t_a, and t_b meeting E_i=0 are the parameters that create two soft landing points, i.e., the design/control parameters capable of producing the soft landing. Since there are seven variables, the infinite solution sets mathematically including the negative mass and/or complex number we cannot choose as the practical parameters exist in the range of eq. (2). The practical parameters must be found empirically within the range of 0.5<=m_1, m_2<=4.0, 100<=k<=2500 for the design parameters and 0.01<=h_a, h_b<=0.4, 0.01<=t_a, t_b<=1.0 for the control parameters. The shaft friction σ was set to 5N, as well in Section 2.

As an example, we obtain the design/control parameters using Newton Raphson method (NRM), when the height of the first riser is H_a=0.05 [m] and the height of the second riser is H_b=0.15 [m]. Since, in practical terms, it is the most difficult to fabricate springs with arbitrary spring constants, k, let k be fixed.

Then, the variables, h_i, and t_i, are obtained from m_1 and m_2, because h_i is the function of m_1 and m_2, while t_i is the function of m_1, m_2, and h_i, and E_i is the function of m_1, m_2, h_i, and t_i. Hence, the updated equation for NRM with variables of m_1 and m_2 is described as

$$ {}^{l}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = {}^{l-1}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} - {}^{l-1}\begin{bmatrix} \dfrac{\partial E_a}{\partial m_1} & \dfrac{\partial E_a}{\partial m_2} \\ \dfrac{\partial E_b}{\partial m_1} & \dfrac{\partial E_b}{\partial m_2} \end{bmatrix}^{-1} {}^{l-1}\begin{bmatrix} E_a \\ E_b \end{bmatrix} \qquad (9) $$

where l is the iteration number, k was set to 1500N/m, and n_a and n_b were set to 1 and 2, respectively, as typical parameters. The iterative calculation was performed from the initial values of m_1=1.50 and m_2=1.00. After 10 iterations, the approximate solution, m_1=0.8052, m_2=0.5461, h_a=0.02616, h_b=0.04433, t_a=0.1277, t_b=0.2253 were obtained. The errors were E_a=-4.3*10^{-14} and E_b=3.6*10^{-14}. Using these design/control parameters, the landing height, the vertical velocity, the vertical acceleration, and the vertical jerk for two riser heights were z_{2a}=5.00*10^{-2}, $\dot{z}_{2a}$=2.78*10^{-16}, $\ddot{z}_{2a}$=-8.52*10^{-14}, $\dddot{z}_{2a}$=-9.36*10^{2}, z_{2b}=1.50*10^{-1}, $\dot{z}_{2b}$=-2.22*10^{-16}, $\ddot{z}_{2b}$=-6.75*10^{-14}, and $\dddot{z}_{2b}$=-1.94*10^{3}, respectively. This is almost the exact solution and has sufficient accuracy to ensure the practical performance of soft landings. **Figure 4** shows the hopping trajectories of Bodies 1 and 2 for two riser heights, H_a=0.05 and H_b=0.15. Here, in order to design a spring contraction that is less than 25% of its natural length, the natural length of the spring was set to 0.2m. We found that two soft landing points were created at the desired riser heights, "A" and "B".

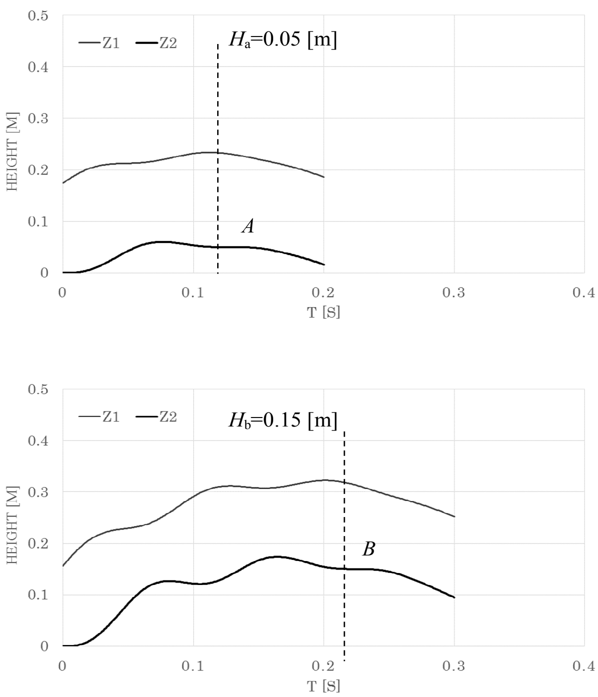

Fig.4 Hopping trajectories of Bodies 1 and 2 for riser heights of H_a=0.05 and H_b=0.15 (k=1500)

As another example, when k=2000, H_a=0.05, and H_b=0.15, after 10 iterations, we also obtained the approximate solution of m_1=2.1113, m_2=0.5461, h_a=0.01962, h_b=0.03324, t_a=0.1277, t_b=0.2253, E_a=7.1*10^{-15}, and E_b=8.0*10^{-15}. Using these design/control parameters, the landing height, the vertical velocity, the vertical acceleration, and the vertical jerk for two riser heights were z_{2a}=5.00*10^{-2}, $\dot{z}_{2a}$=1.11*10^{-16}, $\ddot{z}_{2a}$=1.78*10^{-14}, $\dddot{z}_{2a}$=-9.36*10^{2}, z_{2b}=1.50*10^{-1}, $\dot{z}_{2b}$=-6.11*10^{-16}, $\ddot{z}_{2b}$=-8.88*10^{-15}, and $\dddot{z}_{2b}$=-1.94*10^{3}, respectively. **Figure 5** shows the hopping trajectories of Bodies 1 and 2 for two riser heights, H_a=0.05 and H_b=0.15. We also found that two soft landing points were created at the desired riser heights, "A" and "B". Since the upper body mass in the case of k=2000 is heavier than that in the case of k=1500, the amplitude of the upper body in the case of k=2000 is smaller than that in the case of k=1500. Since, in the case of X=2, the solution space is not continuous but wide, we can create and choose various soft landing points, i.e., design/control parameters.

In this section, although we fixed the spring constant among seven variables, other variables can be fixed as well. Moreover, even though other conditions such as energy minimization, reeling torque minimization, and total mass minimization can be added, the discrete solution space narrows.

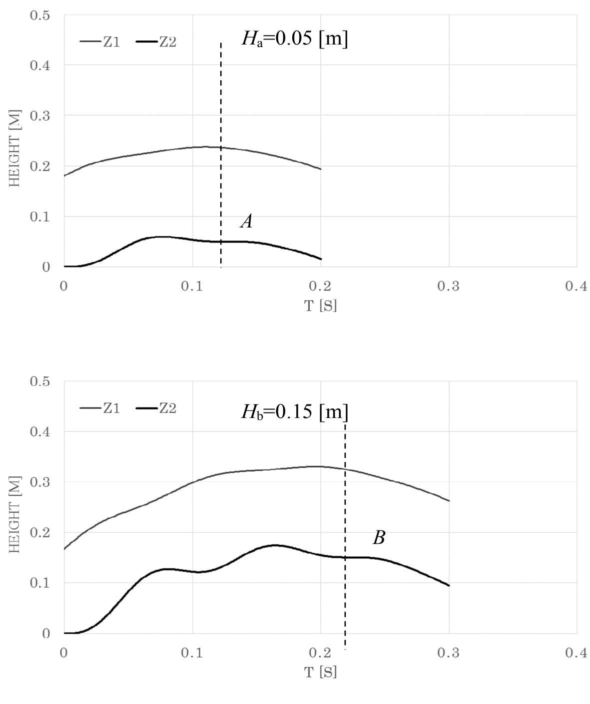

Fig.5 Hopping trajectories of Bodies 1 and 2 for riser heights of H_a=0.05 and H_b=0.15 (k=2000)

5 Three kinds of soft landing points (X=3)

Let three different riser heights be H_a, H_b, and H_c ($H_a \neq H_b$, $H_b \neq H_c$, $H_c \neq H_a$) and let the amplitude constants after takeoff and the landing times be h_a, h_b, h_c, t_a, t_b, and t_c, nine conditions, eq. (8) with i=a, b, c, are obtained. In this case, since the number of the unknown variable corresponds with the condition number, the solution set is uniquely determined. Here, we try to obtain the practical solution set (nine variables: m_1, m_2, k, h_a, h_b, h_c, t_a, t_b, and t_c) from among the sets of solution meeting E_i=0. Since the solution set is uniquely determined and additionally, since the error function is not continuous (awfully discrete), it is difficult to obtain the solution set by the iterative algorithm using continuous differential equations such as NRM. Thus, we used the simplex algorithm [9] with the error function

$$E = \sum_i E_i^2 \tag{10}$$

and obtained the set of solution minimizing the function, E.

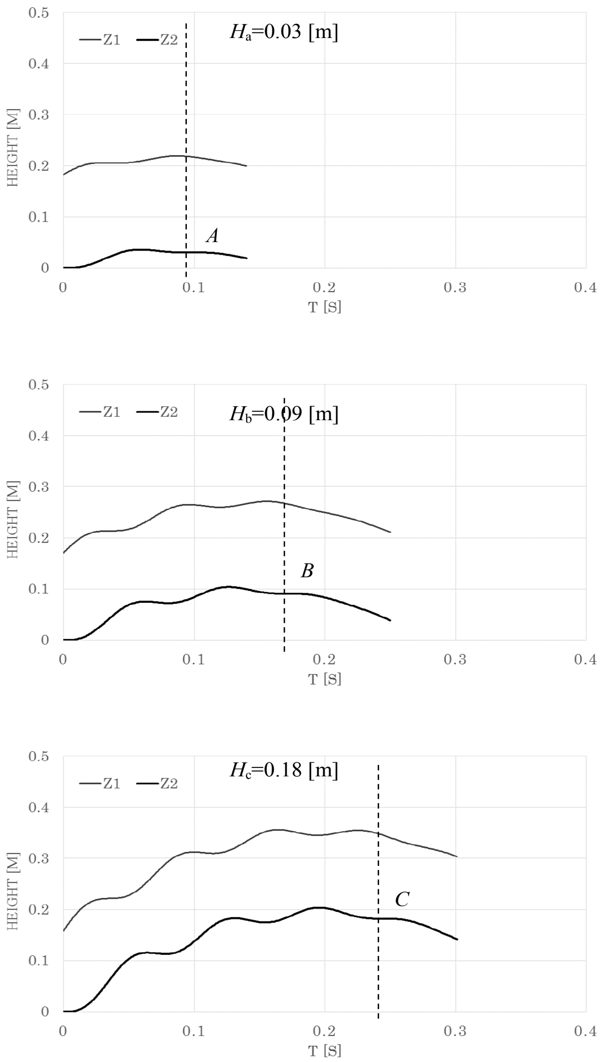

Fig.6 Hopping trajectories of Bodies 1 and 2 for riser heights of H_a=0.03, H_b=0.09, and H_c=0.18

As an example, we obtain the design/control parameters among above practical range, when the heights of riser are H_a=0.03, H_b=0.09, and H_c=0.18. For use as typical parameters, n_a, n_b, and n_c were set to 1, 2 and 3, respectively. After 200 iterations, we obtained the approximate solution of m_1=0.5900, m_2=0.5313, k=2206, h_a=0.0175, h_b=0.0296, h_c=0.0417, t_a=0.0973, t_b=0.1720, and t_c=0.2442. The error was E=9.96*10^{-3}. Using these design/control parameters, the landing height, the vertical velocity, the vertical acceleration, the vertical jerk for three riser heights were z_{2a}=3.00*10^{-2}, $\dot{z}_{2a}$=9.00*10^{-4}, $\ddot{z}_{2a}$=-1.05*10^{-1}, $\dddot{z}_{2a}$=-1.14*10^{3}, z_{2b}=9.00*10^{-2}, $\dot{z}_{2b}$=-5.96*10^{-4}, $\ddot{z}_{2b}$=-1.45*10^{-1}, $\dddot{z}_{2b}$=-2.41*10^{3} z_{2c}=1.80*10^{-1}, $\dot{z}_{2c}$=1.53*10^{-4}, $\ddot{z}_{2c}$=-5.57*10^{-2}, and $\dddot{z}_{2c}$=-3.55*10^{3}, respectively. Since the errors of the vertical velocity and acceleration at the landing point are less than 1mm/s and 0.01G, we can choose this solution as the practical soft landing point, but the error is large compared with that of X=2. When we solved this equation as an inverse problem, we obtained the almost exact solution of E<10^{-12} when H_a=0.02997, H_b=0.09003, and H_c=0.17999. However, we also found that it was very difficult to find a practical solution in this case, because the solution space is discrete and very narrow, and the conditions are awfully discrete and even multimodal. **Figure 6** shows the hopping trajectories of Bodies 1 and 2 for three riser heights, H_a=0.03, H_b=0.09, and H_c=0.18. We also found that the soft landing points were created at the riser heights, "A", "B", and "C".

6 Conclusions

In this paper, as part of our study aimed at the development of a fast stair-climbing robot that lands softly, despite multiple riser heights, by changing its control parameters, we clarified that three soft landing points could be created from the condition of the existence obtained by equations of motion. More specifically, we mathematically showed that the solution sets of the design/control parameters for realizing two soft landing points existed infinitely, and that the solution set for realizing three soft landing points was determined uniquely. Moreover, we showed an example of a feasible solution among the practical range.

Our future work is to clarify the theoretical relationship between the combination of riser heights and the existence characteristics of the solution and to demonstrate practical soft landings using the hardware with the obtained design/control parameters by discussing the required horizontal motion.

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Received on December 15, 2016
Accepted on March 29, 2017