# Soft Landing Condition for Stair-Climbing Robot with Hopping Mechanism

**Akira KAMEI*[1], Masaya JINNO*[1], Naoya CHISHIMA*[1], Kenji SATO*[2], and Koki KIKUCHI*[1]**

*1 The Department of Advanced Robotics, Chiba Institute of Technology
2-17-1 Tsudanuma, Narashino, Chiba 275-0016, Japan
kikut@ieee.org

*2 Shibaura Holdings, Inc.
1-2-1 Nishiura, Funabashi, Chiba, 273-0017, Japan
ke-sato@shivaura.co.jp

## Abstract

In this paper, we mathematically analyze the feasibility of soft landing for a fast stair-climbing robot with a hopping mechanism. The robot, consisting of two bodies connected by springs and a wire, hops by releasing energy stored in the springs and travels quickly using wheels mounted on its lower body. The trajectories of the bodies during hopping depend on the design and control parameters. Although this robot realizes fast stair climbing and almost 0G soft landing without complex control, it is difficult to obtain the parameters required for soft landing. Here we investigate the relationship between the existence of a 0G soft landing condition and the design and control parameters using the equation of motion, and we theorehically clarify the characteristics of soft landing.

**Keywords:** stair-climbing robot, vibration, soft landing

## 1 Introduction

With the aim of achieving traversability over rough terrain, various mechanisms have been proposed and developed [1]-[5]. A wheel-type vehicle can move quickly and quite stably but cannot overcome obstacles higher than its wheel radius. A crawler-type vehicle can negotiate terrain with acertain degree of roughness but is slow and impairs the ground. A legged vehicle can climb over higher obstacles than other types of vehicles but requires complex control. Although hybrids of these mechanisms or a robot composed of several bodies, such as a snake robot, have also been proposed, most are not practical in terms of simple and easy operation such as surveillance in an office, because of the design and control costs. This is because they are expensive general-purpose vehicles designed to negotiate various rough terrains such as off-road surfaces and steps. We believe that a more specialized mechanism should be developed for limited operation environments, such as standardized stairs in an office building.

From this viewpoint, we have proposed and developed a wheel-type robot with a hopping mechanism, that quickly climbs stairs by vibration and lands without impact, i.e., at 0G [6]-[8]. The robot, consisting of two bodies connected by springs and a wire, hops by releasing energy stored in the springs and travels quickly using wheels mounted on its lower body. The trajectories of the bodies during hopping depend on (a) design parameters, such as the reduced mass of the two bodies, the mass ratio, and the spring constant, and (b) control parameters, such as the initial contraction of the spring and the horizontal velocity. Although this mechanism realizes fast stair climbing and almost 0G soft landing without complex control, it is difficult to obtain the design and control parameters required for soft landing. Thus far, we have found the parameters using a stochastic searching scheme, i.e., a genetic algorithm (GA). In this paper, we investigate the relationship between the existence of a 0G soft-landing condition and the design and control parameters using the equation of motion, and we theoretically clarify the characteristics of soft landing.

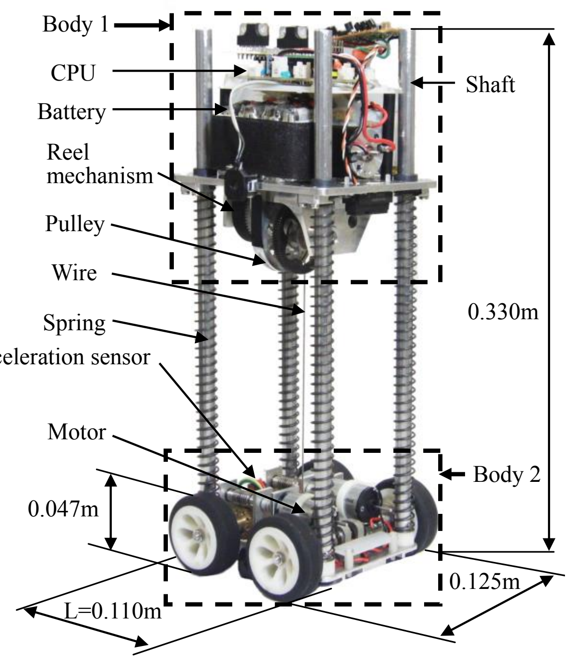


**Fig. 1 Overview of stair-climbing robot**

## 2 Mathematical model of the robot

**Figure 1** shows the developed stair-climbing robot. The robot consists of upper and lower bodies (Bodies 1 and 2) connected by four shafts, four springs, and a wire. The upper body has a mechanism for reeling the wire, a CPU for control, and a battery, and the lower body has four wheels and two motors, enabling it to travel horizontally. The robot travels quickly using the wheels, hops to a considerable height by releasing the energy stored in the springs, and lands softly by canceling the downword velocity of the lower body using the ascending velocity of the vibration.

**Figure 2** shows a simplified mathematical 2D model for the stair climbing robot shown in **Fig.1**. From this, the equation of motion is described as the two degrees of freedom (2DOF) spring mass system

$$\begin{aligned}&(m_1+m_2)\ddot{x}_1=(m_1+m_2)\ddot{x}_2=f_x\\&m_1\ddot{z}_1=k(z_2-z_1)-m_1g-T_w-\mu_tF_f\\&m_2\ddot{z}_2=k(z_1-z_2)-m_2g+T_w+\mu_tF_f+N\end{aligned}\quad,\qquad(1)$$

where ($x_i$, $z_i$) is a coordinate system for the $i^{th}$ body, $m_i$ is the mass, $k$ is a spring constant, $f_x$ is the motor force for horizontal travel, $\mu_t F_f$ is the friction of the shaft, $N$ is the reaction of the ground, $g$ is a gravitation constant (=9.8m/s$^2$), and $T_w$ is the wire tension. Note that the posture of the robot is neglected in this study (that is $x_1$=$x_2$) and the natural length of the spring means $z_1$-$z_2$=0. The robot stores the spring energy by contracting the wire using the reeling mechanism and hops by releasing it. Thus, the robot hops, that is, the lower body takes off, if and only if it meets the following conditional expression

$$m_2g<k(z_1-z_2)+T_w-\mu_tF_f\,. \qquad (2)$$

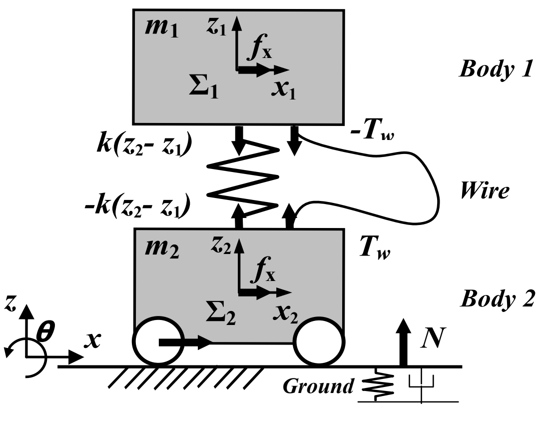


**Fig.2 Two-dimensional mathematical model**

## 3 Definition of soft landing

In previous study, we have defined a "soft landing" as a special landing in which the vertical velocity of the lower body is almost zero ($\dot{z}_2 \cong 0$), the acceleration is almost zero ($\ddot{z}_2 \cong 0$), and the jerk is zero or negative ($\dddot{z}_2 \le 0$) at the landing height ($z_2 = H$) [6]-[8]. Mathematically, this is a stationary and inflection point. **Figure 3** shows two typical examples of soft landing, Cases A and B. The bold lines are the trajectories of the center of mass (COM) in Case A with design and control parameters of $m_1$=2.00[kg], $m_2$=1.00[kg], $k$=1400[N/m], and $h$=0.09[m]. On the other hand, the dashed lines are the trajectories of the COM in Case B with design and control parameters of $m_1$=2.00[kg], $m_2$=1.00[kg], $k$=2000[N/m], and $h$=0.11[m]. Here, $h$ is the initial contraction of the spring. We can see that soft landing occurs at points A and B in **Fig.3**. In references [6]-[8], it is shown that the angular frequency during hopping is determined by the spring constant and the reduced mass of two bodies, and the vibration amplitude is determined by the upper to lower mass ratio and the initial contraction of the spring. Therefore, the soft landing point can be controlled via (a) design parameters, such as the reduced mass, the mass ratio, and the spring constant, and (b) control parameters, such as the initial contraction of the spring and the horizontal velocity. Once these parameters are determined, this mechanism realizes fast and impact-free stair climbing without complex control in an environment with stairs consisting of standardized riser and tread.

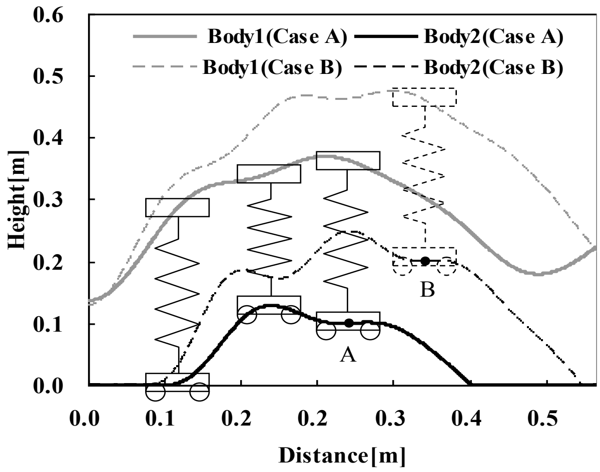


**Fig.3 Two typical examples of trajectories of bodies 1 and 2 during hopping**

## 4 Feasibility of soft landing

To investigate the feasibility of soft landing, i.e., the existence of a soft-landing point, we mathematically redefine a "soft landing" as a special landing in which the vertical velocity of the lower body is zero ($\dot{z}_2 = 0$), the acceleration is zero ($\ddot{z}_2 = 0$), and the jerk is zero or negative ($\dddot{z}_2 \le 0$) at the landing height ($z_2 = H$). Here, from the viewpoint of the shaft friction, we discuss the existence of a soft-landing point, that is, mathematically exact solution, using the equation of motion (eq.(1)) and the conditional expression for takeoff (eq.(2)). Note that we do not control the wire tension during hopping ($N$=0) for simplification of the motion control ($T_w$=0) in this paper.

### 4.1 In the case without friction ($\mu_t F_f = 0$)

Focusing on the vertical motion in eq.(1), the body trajectories during hopping ($0 \le z_2$, $N$=0) are described as

$$
\begin{aligned}
z_1 &= \frac{Mh}{m_1}\sin(\omega t+\phi) - \frac{1}{2}g(t-T)^2 + C \\
z_2 &= -\frac{Mh}{m_2}\sin(\omega t+\phi) - \frac{1}{2}g(t-T)^2 + C \,, \\
&\omega = \sqrt{k/M}\,, \quad M = m_1 m_2/(m_1+m_2)
\end{aligned}
\tag{3}
$$

where $\phi$, $T$, and $C$ are constants determined by the initial conditions, $\omega$ is the angular frequency, and $M$ is the reduced mass. Here, the first and second terms on the right side represent the vibration of the 2DOF system and the parabolic motion of the COM of the robot, respectively. Then, $T$ expresses the axis of symmetry of the parabolic motion. Assuming that the takeoff time is zero ($t$=0), the position and velocity of the lower body are zero at this time ($z_2 = 0, \dot{z}_2 = 0$). Since the spring force is continuous and the takeoff condition is expressed by eq.(2), the above constants are simply solved as

$$
\begin{aligned}
\sin\phi &= \frac{m_2 g}{kh} \\
T &= \frac{Mh}{m_2 g}\omega\sqrt{1-(\frac{m_2 g}{kh})^2} \,. \\
C &= \frac{Mh}{2m_2}(\frac{hk}{m_2 g} + \frac{m_2 g}{hk})
\end{aligned}
\tag{4}
$$

On the other hand, letting the instantaneous time of soft landing be $t$=$t_{\mathrm{land}}$, from the acceleration condition of the soft landing ($\ddot{z}_2(t_{land}) = 0$), we obtain

$$
\frac{hk}{m_2}\sin(\omega t_{land}+\phi) - g = 0 \,. \tag{5}
$$

Thus, using the first equation of eq.(4) and eq.(5), we also obtain

$$
\omega t_{land} = 2n\pi,\ -2\phi + (2n+1)\pi \,, \tag{6}
$$

where $n$ is a positive integer denoting the iteration number of the vibration. Moreover, from the jerk condition of the soft landing ($\dddot{z}_2(t_{land}) \le 0$), we have

$$
\cos(\omega t_{land}+\phi) \le 0 \,. \tag{7}
$$

**Case Ia:** $\omega t_{land} = 2n\pi$

From the velocity condition of the soft landing ($\dot{z}_2(t_{land}) = 0$) and eq.(5), we obtain

$$
\frac{Mh}{m_2}\omega\sqrt{1-(\frac{gm_2}{hk})^2} - n\pi g\sqrt{\frac{M}{k}} = 0 \,. \tag{8}
$$

Here, although the substitution of eq.(5) and eq.(8) into the second equation of eq.(3) gives $z_2(t_{land}) = 0$, this is inconsistent with the position condition of the soft landing, $z_2(t_{land}) = H$. Hence, an exact solution (0<$m_1$, 0<$m_2$, 0<$k$, 0<$h$) for the soft landing does not exist in Case Ia.

**Case Ib:** $\omega t_{land} = -2\phi + (2n+1)\pi$

Substituting eq.(5) into the velocity condition of the soft landing ($\dot{z}_2(t_{land}) = 0$), we have

$$
-2\phi + (2n+1)\pi = 0 \,. \tag{9}
$$

However, $\omega t_{land} = 0$, that is, either the angular frequency is zero or the soft landing time is zero, which is not the actual solution. Hence, an exact solution for the soft landing does not exist in Case Ib.

From the above, we can conclude that a soft-landing point does not exist in the case without friction; hence, a perfect soft landing is not realized.

**4.2 In the case with friction ($\mu_t F_f \ne 0$)**

In this section, we take into account the shaft friction and discuss the existence of a soft-landing point by assuming the shaft friction to be Coulomb friction:

$$
\mu_t F_f = \begin{cases} \sigma & if\ \ 0 < \dot{z}_1 - \dot{z}_2 \\ 0 & if\ \ \dot{z}_1 - \dot{z}_2 = 0 \\ -\sigma & if\ \ \dot{z}_1 - \dot{z}_2 < 0 \end{cases} \,, \tag{10}
$$

where $\sigma$ is a positive constant and the sign of the friction depends on the relative velocity between the lower and upper bodies. As indicated in the previous section, assuming that the wire is not controlled during hopping ($T_{\mathrm{w}}$=0), the trajectories of the lower and upper bodies are described for two cases depending on the sign of the friction. The gray and white areas in **Fig. 4** show regions with positive and negative signs for the friction, respectively.

In the case of $0 \le \dot{z}_1 - \dot{z}_2 = (h - 4n\sigma/k)\omega\cos(\omega t+\phi)$:
When the spring is expanding, i.e., $-\pi/2 \le \omega t + \phi - 2n\pi \le \pi/2$,

$$
\begin{aligned}
z_1 = &\frac{M}{m_1}(h - \frac{4n\sigma}{k})\sin(\omega t+\phi) \\
&-\frac{1}{2}g(t-T)^2 + \frac{Mg}{2k}[(\frac{kh}{m_2 g})^2+1] - \frac{\sigma}{k}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
z_2 = &-\frac{M}{m_2}(h - \frac{4n\sigma}{k})\sin(\omega t+\phi) \\
&-\frac{1}{2}g(t-T)^2 + \frac{Mg}{2k}[(\frac{kh}{m_2 g})^2+1]
\end{aligned}
\,. \tag{12}
$$

In the case of
$0 > \dot{z}_1 - \dot{z}_2 = [h - (4n+2)\sigma/k]\omega\cos(\omega t+\phi)$: When the spring is contracting, i.e., $\pi/2 \le \omega t + \phi - 2n\pi \le 3\pi/2$,

$$z_1 = \frac{M}{m_1}[h - \frac{(4n+2)\sigma}{k}]\sin(\omega t+\phi) - \frac{1}{2}g(t-T)^2 + \frac{Mg}{2k}[(\frac{kh}{m_2 g})^2 + 1] - \frac{m_1 - m_2}{m_1 + m_2}\frac{\sigma}{k} \tag{13}$$

$$z_2 = -\frac{M}{m_2}[h - \frac{(4n+2)\sigma}{k}]\sin(\omega t+\phi) - \frac{1}{2}g(t-T)^2 + \frac{Mg}{2k}[(\frac{kh}{m_2 g})^2 + 1] - \frac{2m_1}{m_1 + m_2}\frac{\sigma}{k} \,. \tag{14}$$

Here, $\phi$, $T$, and $C$ correspond to the values in section 4.1, but $h$ is different from that in case I at the initial contraction of the spring. Next, we discuss the existence of a soft landing point for the following two cases.

**Case IIa:** $-\pi/2 \le \omega t_{land} + \phi - 2n\pi \le \pi/2$
In this case, the acceleration and the jerk at the moment of the soft landing ($t_{land} = 0$) are obtained from eq.(12) as

$$\ddot{z}_2(t_{land}) = \frac{k}{m_2}(h - \frac{4n\sigma}{k})\sin(\omega t_{land} + \phi) - g = 0 \tag{15}$$

$$\dddot{z}_2(t_{land}) = \frac{k}{m_2}(h - \frac{4n\sigma}{k})\omega\cos(\omega t_{land} + \phi) \,\cdot \tag{16}$$

Here, since the amplitude of the vibration in eqs.(11) and (12) must be positive and the condition in case IIa is apparently $0 \le \cos(\omega t_{land} + \phi)$, the sign of eq.(16) is positive ($0 \le \dddot{z}_2(t_{land})$). From this, the jerk condition of the soft landing, $\dddot{z}_2(t_{land}) \le 0$, holds if and only if $\cos(\omega t_{land} + \phi) = 0$. Hence, $\sin(\omega t_{land} + \phi) = 1$ is obtained because $0 < \sin(\omega t_{land} + \phi)$ from eq.(15). Substituting this equation into the velocity condition of the soft landing ($\dot{z}_2(t_{land}) = 0$),

$$t_{land} = \sqrt{\frac{M}{k}}\sqrt{(\frac{kh}{m_2 g})^2 - 1} \tag{17}$$

is obtained. Thus, the position condition of the soft landing ($z_2(t_{land}) = H$) is

$$-\frac{M}{m_2}(h - \frac{4n\sigma}{k}) + \frac{Mg}{2k}[(\frac{kh}{m_2 g})^2 + 1] = H \,\cdot \tag{18}$$

Although the exact solution ($0<m_1$, $0<m_2$, $0<k$, $0<h$) satisfying these equations comprises the parameters realizing the soft landing, since the point at the landing time is the vertex of the parabolic motion ($t_{land} = T$), we cannot choose such a position as the actual soft landing point. This is because the robot has a physical body length and must glide above the tread by at least its body length to land both the front and rear wheels on the stairs simultaneously.

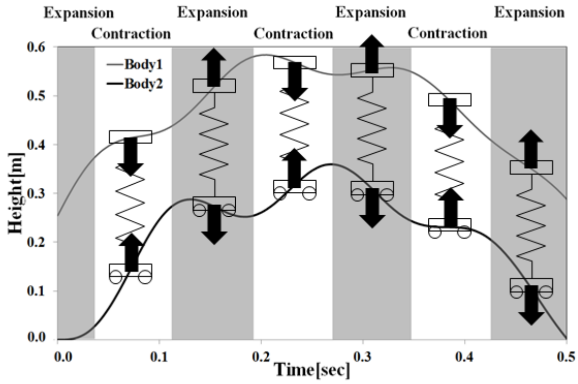


**Fig.4 An example of regions with positive and negative signs for the friction**

**Case IIb:** $\pi/2 < \omega t_{land} + \phi - 2n\pi < 3\pi/2$
In this case, the acceleration and the jerk at the moment of the soft landing ($t_{land} = 0$) are obtained from eq.(14) as

$$\ddot{z}_2(t_{land}) = \frac{k}{m_2}[h - \frac{(4n+2)\sigma}{k}]\sin(\omega t_{land} + \phi) - g = 0 \tag{19}$$

$$\dddot{z}_2(t_{land}) = \frac{k}{m_2}[h - \frac{(4n+2)\sigma}{k}]\omega\cos(\omega t_{land} + \phi) \,\cdot \tag{20}$$

Here, since the amplitude of the vibration must be positive from eqs.(13) and (14) ($0 < h - (4n+2)\sigma/k$) and the condition of case IIb is $\cos(\omega t_{land} + \phi) < 0$, eq.(20) becomes $\dddot{z}_2(t_{land}) < 0$. Thus, this case satisfies the jerk condition of the soft landing. Next, from eq.(19) and the velocity condition of the soft landing ($\dot{z}_2(t_{land}) = 0$),

$$t_{land} = \frac{M\omega}{m_2 g}[h - \frac{(4n+2)\sigma}{k}]\sqrt{1 - (\frac{m_2 g}{kh - (4n+2)\sigma})^2} + T \tag{21}$$

is obtained. Then, from the position condition of the soft landing ($z_2(t_{land}) = H$),

$$\frac{M\sigma}{m_2}[\frac{h(4n+2)}{m_2 g} - \frac{(4n+2)^2\sigma}{2m_2 gk} - \frac{2}{k}] = H \tag{22}$$

is also obtained. Hence, the exact solution ($0<m_1$, $0<m_2$, $0<k$, $0<h$) satisfying eqs. (19), (21), and (22) comprises the parameters realizing the soft landing.

From this, we can conclude that an exact solution realizing the soft landing exists under spring contraction in the presence of friction.

On the other hand, since the first term on the right side of eq.(21) is positive, we can see that the soft-landing point exists after the vertex of the parabolic motion. As indicated in the previous discussion, since the robot must glide above the tread by at least the body length, assuming that horizontal velocity $\dot{x}_2$ is constant during hopping, the following condition is obtained

$$L \le 2\dot{x}_2 \frac{M\omega}{m_2 g}[h - \frac{(4n+2)\sigma}{k}]\sqrt{1-(\frac{m_2 g}{kh-(4n+2)\sigma})^2} \; . \tag{23}$$

This equation determines the minimum horizontal velocity, and the condition that the horizontal traveling distance must be less than the tread limits the maximum horizontal velocity. From this, the control range of the horizontal traveling velocity ($\dot{x}_2$) is determined.

In this paper, we discussed the feasibility of soft landing by passive control after takeoff. If we take into account the control of the wire tension during hopping, a soft landing point exists in all cases (cases Ia, Ib, IIa, and IIb). However, since this implies force control by the positive wire tension ($0 \le T_w$) and requires a force sensor with high sampling frequency, the implementation remains somewhat difficult from the viewpoint of design and control costs. Such implementation will be an interesting future work.

## 5 Example of soft landing

In this section, we show an example of soft landing by obtaining the exact solution. The design parameters are the masses of the lower and upper bodies, $m_1$ and $m_2$, respectively, and the spring constant, $k$, and the control parameters are the initial spring contraction variable, $h$, the vibration number, $n$, and the landing time, $t_{land}$. Since these six parameters are solved using three equations, i.e., eqs. (19), (21), and (22), and two conditions, i.e., the condition of case IIb and the condition that the amplitude of the vibration is positive, an infinite number of exact solutions exist. Here, we introduce the exact solution (**Table 1**) approximately corresponding to the experimental parameters in previous works [6]-[8]. Note that the riser $H$ was set to 0.2m for common stairs, the shaft friction was 5N from experimental results, and the tread length and horizontal velocity were neglected for simplicity of discussion.

**Figures 5-8** respectively show the vertical trajectories of the lower and upper bodies, the vertical velocities, the vertical accelerations, and the vertical jerks over time. Here, the gray and white areas respectively show the expansion and contraction phases of the springs. We can see that a soft-landing point appears on the dashed line in the white area, where the landing height is 0.2m, the vertical velocity is zero, the vertical acceleration is zero, and the vertical jerk is less than zero, and that the friction attenuates the amplitude of the vibration. In this paper, we focused on the shaft friction as one of the attenuation terms and discussed the existence of a soft-landing point. In future works, we intend to clarify the essential mechanism determining the existence of a soft-landing point, for example, the damper.

**Table 1 Design and control parameters**

| $m_1$[kg] | $m_2$[kg] | $k$[N/m] | $h$[m] | $n$ | $t_{land}$ [s] |
|---|---|---|---|---|---|
| 1.535 | 1.376 | 1000 | 0.13 | 2 | 0.4158 |

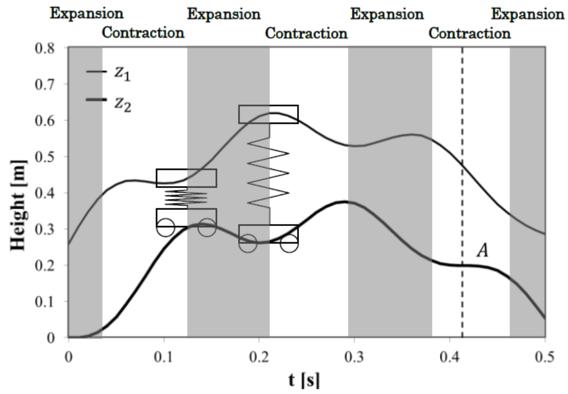


**Fig.5 Trajectories of bodies 1 and 2 during hopping**

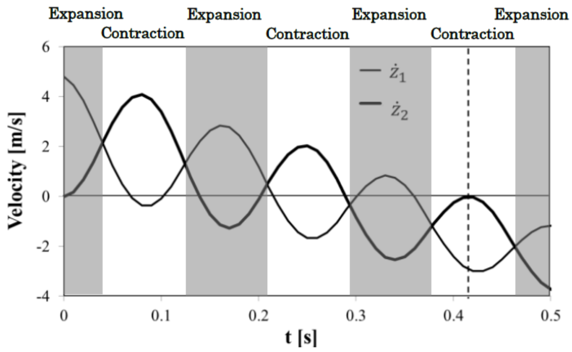


**Fig.6 Velocities of bodies 1 and 2 during hopping**

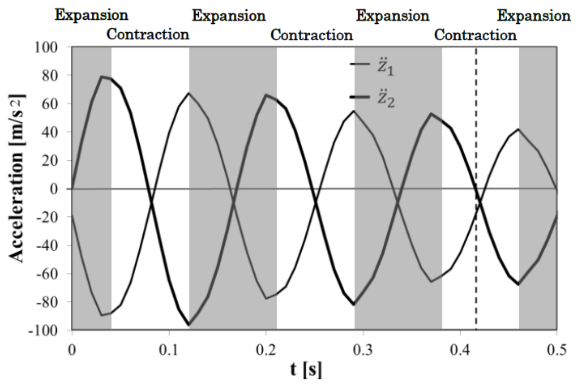


**Fig.7 Accelerations of bodies 1 and 2 during hopping**

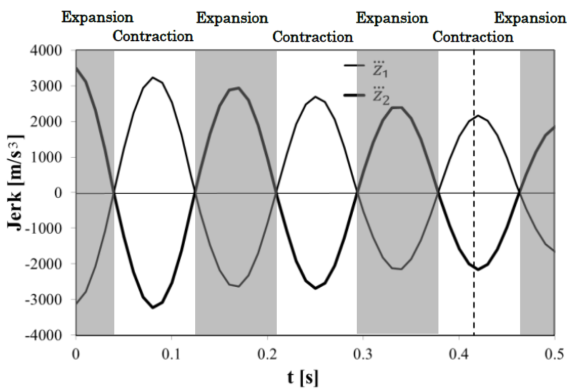


**Fig.8 Jerks of bodies 1 and 2 during hopping**

## 6 Conclusions

In this paper, to realize the 0G soft landing of a fast stair-climbing robot with a hopping mechanism, we analyzed the existence of a soft-landing point, i.e., the existence of an exact solution, using the equation of motion from the viewpoint of shaft friction as one of the attenuation terms. Additionally, we discussed the feasibility of soft landing.

A future work is to investigate the robustness of soft landing against sensor and actuator errors.

## References


[1] Hirose, S., Sensu, T. and Aoki, S., "The TAQT Carrier: A Practical Terrain-Adaptive Quadru-Track Carrier Robot," Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, (1992), pp.2068-2073.

[2] Saranli, U., Buehler, M. and Koditschek, D. E., "RHex: A Simple and Highly Mobile Hexapod Robot," International Journal of Robotics Research, Vol.20, No.7, (2001), pp.616-628.

[3] Sugahara, Y., Carbone, G., Hashimoto, K., Ceccarelli, M., Lim, H. and Takanishi, A., "Experimental Stiffness Measurement of WL-16RII Biped Walking Vehicle during Walking Operation," Journal of Robotics and Mechatronics, Vol.19,No.3, (2007), pp.272-280.

[4] Yim, M. H., Homans, S. B. and Roufas, K. D., "Climbing with Snake-like Robots," IFAC Workshop on Mobile Robot Technology, (2001), pp.21-22.

[5] Nakajima, S., Nakano, E. and Takahashi, T., "Motion Control Technique for Practical Use of a Leg-Wheel Robot on Unknown Outdoor Rough Terrains," Proceedings of IEEE/RSJ International conference on Intelligent Robots and Systems, Vol.1, (2004), pp.1353-1358.

[6] Sakaguchi, K., Sudo, T., Bushida, N., Chiba, Y., Asai, Y. and Kikuchi, K., "Wheel-Based Stair-climbing Robot with Hopping Mechanism – Fast Stair-climbing and Soft-landing by Vibration of 2-DOF system –," The Japan Society of Mechanical Engineers, Journal of Robotics and Mechatronics, Vol.19, No.3, (2007), pp.258-263.

[7] Asai, Y., Chiba, Y., Sakaguchi, K., Sudo, T., Bushida, N., Otsuka, H., Saito, Y. and Kikuchi, K., "Wheel-based Stair-climbing Robot with Hopping Mechanism - Demonstration of Continuous Stair Climbing Using Vibration –," The Japan Society of Mechanical Engineers, Journal of Robotics and Mechatronics, Vol.20, No.2, (2008), pp.221-227.

[8] Kikuchi, K., Sakaguchi, K., Sudo, T., Bushida, N., Chiba, Y. and Asai, Y., "A Study on Wheel-based Stair-climbing robot with Hopping Mechanism", Mechanical Systems and Signal Processing, Elsevier, Vol.22, No.6, (2008), pp.1316-1326.



Received on October 31, 2013

Accepted on January, 28, 2014