The result shows the best performance, 64.5%, of the solve this problem. Second, our proposed method corresponds to actual sensation votes. Because most 6.3 Discussion 43.4%

occupant's vote. 'ocupant'
network-based method mentioned before. In this paper, to PMVs. Even the large difference between vote and 7 days on August 4th through 10th. The lower graph shows the correlation between the occupant's sensation vote with PMVs derived from the temperature and humidity. These data change probabilistic distribution of occupant's temperature width were shown. After that, using an experimental results. Our method enables to predict how to update probability tables were shown in detail. Finally, probability tables which present the example case, we described detailed procedure of our method can easily be applied to multi-sensor system. Moreover, omitting air-conditioning at that thermal index as PMV does not always function sufficiently. PMV arises from individual difference. This result shows not all sensation votes correspond to thermal indices like PMV. Because most conventional thermal indices did not always correspond to actual sensation votes. Because most conventional thermal indices did not always correspond to actual sensation votes. These data change.

1 Introduction

In recent years, it has been eagerly desirable that patients, medicines and small precision parts should be carried smoothly and quietly by advanced carts with casters. In order to realize the ideal cart with low crashes and vibrations, some casters equipped with damping elements have been developed [1]. On the other hand, although the casters with dampers can restrict the cart vibrations, they had little ability to reduce the crashed acceleration of the cart. Hence we designed a new caster focusing on the center of percussion of the caster, and verified experimentally the effective reduction of the cart accelerations using a cart with the new caster [2], [3]. However, the cart vibrations were not removed effectively by the new caster.

Thus we design an active controlled caster that has a mechanical low-crashed structure based on the idea of center of percussion. First the dynamical model of the cart with a caster is derived to study the caster control for the low crashes and vibrations. Then we show experimental results of the cart with the caster controlled by the acceleration and velocity signals, compared to the simulation results. The proposed control is verified to effectively reduce the impulsive crashes and vibrations of the cart.

Keywords: caster, design, control, dynamics

2 Active caster with low-crash mechanism

2.1 Low-crash mechanism of caster

Figure 1 shows a side view of a swing-arm typed caster with an elastic and viscous element. The elastic and viscous element has usually effects to reduce the cart vibrations, and also has a role to keep the swing arm horizontal. On the other hand, the cart has large crashes through the swing arm when the caster wheel with the swing arm collides with a small bump on a road. It is rather difficult for the momentary crashes to be removed by only the elastic and viscous element. Thus we have proposed a mechanical design based on the center of percussion of the swing arm with a wheel. We summarize the design concept as follows.

The impulsive force from a road to the wheel center P is nearly perpendicular to the line PQ shown in Figure 1 when the caster wheel collides with a small bump. Hence the transmitted impulsive force to the joint center Q is expected to be effectively reduced when the point Q is located on the center of percussion of the whole swing arm against the point P. The design condition is as follows:

\[ \frac{a}{b} = I_{G} \cdot I_{G} \cdot (1) \]

Here Symbols a and b denote the length between the point P and the gravity center G of the swing arm, and the length between the point G and the point Q, respectively. Symbols m and I_G denote the mass of the swing arm and the moment of inertia about the gravity center G, respectively. Applying the above concept based on Equation (1) to the swing arm design, we have

Design of an Active Controlled Caster
Aiming at Cart with Low Crashes/Vibrations

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Abstract

This paper presents experimental and simulational results of a cart with an active controlled caster to reduce the cart crashes and vibrations. First the active controlled caster is introduced, which has a mechanical low-crash structure based on the idea of center of percussion. Next the dynamical model of the cart with the caster is derived to study the caster control for the low crashes and vibrations. Then we show experimental results of the cart with the caster controlled by the acceleration and velocity signals, compared to the simulation results. The proposed control is verified to effectively reduce the impulsive crashes and vibrations of the cart.

Keywords: caster, design, control, dynamics

1 Introduction

In recent years, it has been eagerly desirable that patients, medicines and small precision parts should be carried smoothly and quietly by advanced carts with casters. In order to realize the ideal cart with low crashes and vibrations, some casters equipped with damping elements have been developed [1]. On the other hand, although the casters with dampers can restrict the cart vibrations, they had little ability to reduce the crashed acceleration of the cart. Hence we designed a new caster focusing on the center of percussion of the caster, and verified experimentally the effective reduction of the cart accelerations using a cart with the new caster [2], [3]. However, the cart vibrations were not removed effectively by the new caster.

Thus we design an active controlled caster that has a mechanical low-crashed structure based on the idea of center of percussion. First the dynamical model of the cart with a caster is derived to study the control method for low crashes and vibrations. Next we show experimental results of the cart with the caster controlled by the acceleration and velocity signals, compared to the simulation results. The application of the control design is verified to effectively reduce the impulsive crashes and vibrations of the cart. Finally we in brief discuss the effect of the disturbance control using the transfer functions and Bode gain diagrams of the cart model.

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verified experimentally the effective reduction of the crashes against the cart [2], [3]. However, the residual vibrations of the cart were not removed effectively even by the new designed caster.

2.2 Design of active caster

Here, an active controlled caster is proposed, which has the design concept based on center of percussion introduced in the above section. The support spring is also located to keep the swing arm horizontal between the swing arm and the cart-platform in Figure 2, as shown in Figure 1. A voice coil motor (VCM) is newly set up as a prismatic actuator on behalf of the spring for the cart-load compensation. Points Q and A have a bearing support, a spring for the swing arm and a weight. Two linear bushes are also equipped to make the movement of the VCM smooth. The side and front views are shown in Figure 2.

3 Modeling of cart with active caster

3.1 Motion equation of cart with active caster

Here, we formulate the two-dimensional motion equation of a cart with the active caster. The angle and mass of the cart-platform are assumed to be negligibly small, and to concentrate at one point on the platform, respectively. The caster wheel has also an elastic and viscous property that transmits an external force \( f_e \) from a road when the wheel contacts the road at Point T on itself. Figure 3 shows the side view of the cart, which has a bearing support, a spring for the swing arm and a spring for the cart-load compensation. Points Q and A denote the rotational center of the swing arm and the attached point of the spring for the swing arm, respectively.

\[
\begin{align*}
\theta: & \text{ Swing arm angle from the horizontal road,} \\
M: & \text{Equivalent mass of the cart at the reference point,} \\
m_b: & \text{Mass of the bearing support,} \\
m: & \text{Mass of the swing arm with the wheel,} \\
f_k: & \text{Operational force of the VCM to the cart,} \\
f_c: & \text{Passive force of the load spring to the cart,} \\
f_d: & \text{Passive force of the swing arm’s spring to the cart,} \\
f_g: & \text{Constraint force to the bearing support at Point Q.}
\end{align*}
\]

In addition to the above modeling parameters, the distance between Point G and Point A is defined as Symbol \( a \), and Symbols \( b \) and \( y_G \) are also defined as shown in Figure 1. Figure 4 in particular shows the distance parameters about the swing arm.

![Fig. 4 Distance parameters about swing arm](image)

We derive the motion equation when the angle of the cart-platform from the road is small enough.

Assuming that the external force \( f_e \) itself becomes the transmitted force at the wheel center P, we obtain the vertical Newton’s Equations of the cart with the active caster as follows:

\[
\begin{align*}
M\ddot{y} &= f_e + f_k + f_H - Mg, \quad (2) \\
m_{bG}\ddot{y}_G &= -f_u - f_k + f_H - m_{bG}g, \quad (3) \\
m_{bG}\ddot{y}_Q &= -f_H - f_e - mg. \quad (4)
\end{align*}
\]

Considering that the swing arm rotates, we also obtain the following Euler’s Equation:

\[
I_G\ddot{\theta} = -af_e - ef_H - bf_Q. \quad (5)
\]

Here Symbol \( I_G \) denotes the moment of inertia about Point G. The swing arm angle \( \theta \) is also neglected in the right terms of Equation (5) under the approximate conditions \( \cos \theta \approx 1 \) and \( \sin \theta \approx 0 \).

On the other hand, the heights \( y, y_G \) and the angle \( \theta \) have the following geometrical relationship:

\[
y_G = y_G + b \sin \theta = y_G + b \theta. \quad (6)
\]

Substituting Equation (6) into Equation (5), and eliminating the angle \( \theta \) and the constraint force \( f_G \), we obtain the following formulations:

\[
m_{bG}\ddot{y}_G = -f_u - f_k - f_H + f_e - (m + m_{bG})g. \quad (7)
\]

\[
y_G = \frac{y_G + b \theta}{b}. \quad (8)
\]

\[
m_{bG}\ddot{y}_Q = -f_u - f_k - f_H + f_e - (m + m_{bG})g. \quad (9)
\]

\[
I_G\ddot{\theta} = -af_e - ef_H - bf_Q. \quad (10)
\]

\[
\begin{align*}
\ddot{y} &= \frac{1}{M} \left( f_e + f_k + f_H - Mg \right), \\
\ddot{y}_G &= \frac{1}{m_{bG}} \left( -f_u - f_k + f_H - m_{bG}g \right), \\
\ddot{y}_Q &= \frac{1}{m_{bG}} \left( -f_H - f_e - mg \right).
\end{align*}
\]

\[
I_G\ddot{\theta} = \frac{1}{I_G} \left( -af_e - ef_H - bf_Q \right).
\]
\[
\left( m + \frac{I_e}{b} \right) \ddot{y}_o - \frac{I_e}{b} \dot{y}_o = \left( 1 + \frac{a}{b} \right) f_r - \left( 1 - \frac{e}{b} \right) f_u - mg. \tag{8}
\]

For the reduction of design variables, we define new variables as follows:
\[
l_\alpha = m_e, \quad a = \mu, \quad e = \lambda.
\]

Here Symbol \( m_e \) is considered to denote the equivalent mass of the swing arm about Point Q. Substituting new variables into Equation (8), we obtain the following new expression:
\[
(m + m_o) \ddot{y}_o - m_o \dot{y}_o = (1 + \mu) f_r - (1 - \lambda) f_u - mg. \tag{9}
\]

Equations (2), (7) and (9) are the final motion equations of the cart with the active caster.

### 3.2 Modeling of elastic and viscous force

The passive forces \( f_k \) and \( f_a \) are formulated as linearly elastic and viscous elements as follows:
\[
f_k = K_s \left[ l_{oB} - (y - y_o) \right] - C_s (\dot{y} - \dot{y}_o) \tag{10}
\]
\[
f_a = K_s \left[ l_{oB} - (y - y_a) \right] - C_a (\dot{y} - \dot{y}_a). \tag{11}
\]

Here Symbols \( K_s, C_s \) and \( l_{oB} \) denote the spring constant, the viscous coefficient and the natural length of the load spring, respectively. Similarly Symbols \( K_a, C_a \) and \( l_{oB} \) denote the spring constant, the viscous coefficient and the natural length of the swing arm spring, respectively.

Since Symbol \( y_o \) means the height of Point A from the road, the following relationship is obtained geometrically:
\[
y_a \equiv \left( 1 - \frac{e}{b} \right) y_o + \frac{e}{b} y_o = (1 - \lambda) y_o + \lambda y_o. \tag{12}
\]

Next we describe the external force to the caster wheel from the road. The external force \( f_e \) is also formulated as an elastic and viscous element similar to above passive spring/damper. When the caster wheel runs on a flat road as shown in **Figure 5**. The formulation is as follows:
\[
f_e = K_r (R - y_p) - C_r \dot{y}_p. \tag{13}
\]

Here Symbols \( K_r, C_r \) and \( R \) denote the spring constant, the viscous coefficient and the natural radius of the wheel tire, respectively. On the other hand, the vertically external force \( f_e \) is formulated as follows when the caster wheel collides with a hard and small bump:
\[
f_e = \left( K_r \Delta Z + C_r \Delta \dot{Z} \right) \frac{y_p}{\sqrt{(a - x_p)^2 + y_p^3}}. \tag{14}
\]

Here Symbols \( \Delta Z \) and \( \Delta \dot{Z} \) denote the lapping displacement between the wheel circle and the semicircular bump, and the velocity of that displacement, respectively. In this case, Symbols \( \Delta Z \) and \( \Delta \dot{Z} \) are expressed as follows as shown in **Figure 6**:
\[
\Delta Z = (R + r) - \sqrt{(a - x_p)^2 + y_p^3}
\]

\[
\Delta \dot{Z} = \frac{1}{\sqrt{(a - x_p)^2 + y_p^3}} \left[ (a - x_p) \dot{y}_p - y_p \dot{y}_p \right]. \tag{15}
\]

Here Symbols \( a \) and \( r \) denote the center position and the radius of a semicircular bump, respectively.
on both the cart-platform and the rotational center Q of the swing arm as shown in Figure 2. Two accelerations of \( \ddot{y} \) and \( \ddot{y}_0 \) are acquired by using these two accelerometers. Since the acceleration signals generally have many noises at high frequencies, appropriate low pass filters are added to remove these noises. After applying the low pass filters to the acceleration signals, the velocities are also acquired by time integral of the pass filters are added to remove the signals. The whole feedback signals are obtained by the combination of accelerations and velocities. Figure 7 shows the controller of the cart with an active caster.

4 Experiment of cart with active caster

4.1 Hardware system

First, the hardware components are shown. Figure 8 and Table 1 show the VCM (voice coil motor) for control and the electrical specifications, respectively.

Table 1 Electrical specifications of VCM

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>5.2</td>
</tr>
<tr>
<td>( L )</td>
<td>6.42\times10^{-3}</td>
</tr>
<tr>
<td>( K_f )</td>
<td>15.5</td>
</tr>
<tr>
<td>( K_e )</td>
<td>15.5</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>4.6</td>
</tr>
<tr>
<td>( T )</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2 Mechanical specifications of swing arm

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0.029</td>
</tr>
<tr>
<td>( b )</td>
<td>0.098</td>
</tr>
<tr>
<td>( m )</td>
<td>0.810</td>
</tr>
<tr>
<td>( I_o )</td>
<td>2.62\times10^{-3}</td>
</tr>
</tbody>
</table>

The swing arm is designed as a specific arm based on the idea of center of percussion as mentioned in Chapter 2. Table 2 shows the mechanical specifications of the swing arm. Table 3 shows each mass of the cart with the swing arm. Table 4 shows the specific parameters of spring components. Table 5 shows the parameters of the wheel tire that were measured by experiments.

Table 3 Masses of main parts

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_0 )</td>
<td>1.39</td>
</tr>
<tr>
<td>( M )</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Table 4 Specific parameters of spring components

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{sw} )</td>
<td>9.85\times10^{-2}</td>
</tr>
<tr>
<td>( K_s )</td>
<td>4.00\times10^{-3}</td>
</tr>
<tr>
<td>( C_s )</td>
<td>4</td>
</tr>
<tr>
<td>( l_{ms} )</td>
<td>1.17\times10^{-1}</td>
</tr>
<tr>
<td>( K_p )</td>
<td>3.91\times10^{-3}</td>
</tr>
<tr>
<td>( C_p )</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5 Parameters of wheel tire

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>0.05</td>
</tr>
<tr>
<td>( K_r )</td>
<td>1.20\times10^{-2}</td>
</tr>
<tr>
<td>( C_r )</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 9 shows the experimental setup for the cart with the active caster. The cart is pushed out at 1100[mm/sec] by a linear servo system, and runs on an aluminum plate. A semicircular bump is placed on the plate, which has a radius of 2.5[mm]. All of motion data are acquired by a personal computer. The feedback control is conducted using Labview™ real-time system.

4.2 Experimental result

In this section, we confirm the effects of the active control by experimental results, compared to simulation results. All of the experiments and simulational are conducted under the condition as shown in Figure 9.

Figure 10 shows the acceleration response of Point A on the cart platform in Figure 9 when the caster runs and collides with a small bump without control. The experiment has a good agreement with the simulation results.
from the viewpoint of resonant frequency, and the convergent time to zero is a little different between the experiment and the simulation.

Fig. 10 Acceleration response without control

Next we show some results with feedback control. These experiments and simulations have the low pass filters of 130 [Hz] to remove sensory noises and the cycle time of 1 [msec] to realize real-time control.

Figure 11 shows the controlled result with the feedback signal of only relative velocity. This feedback effect is considered to correspond to the mechanical damping effect between the cart and the bearing support. The residual vibrations are largely repressed, and the first impulsive acceleration is not reduced, compared to that in Figure 10.

Fig. 11 Acceleration response with control

Figure 12 shows the control result with the feedback signals of relative velocity and cart acceleration. The first impulsive acceleration is rather reduced by virtue of the feedback signal of cart acceleration. The acceleration feedback is considered to change the equivalent mass of the cart virtually. Figure 12 reveals that the vibrational components with high frequencies increase compared to Figure 11.

Fig. 12 Acceleration response with control

Figure 13 shows the control result with the additional feedback signal of the velocity of bearing support in order to remove the above vibrations with high frequencies. The simulation results in Figure 13 have better agreements with experimental results than those in Figure 10, because the uncertain elements such as frictions and backlashes are considered to be largely restricted by feedback control.

Fig. 13 Acceleration response with control

Comparing Figure 10 to Figure 13, we confirm that the maximum value of the cart acceleration becomes about 1/4 and the damping time of the residual vibration becomes about 1/5 by virtue of feedback control.

4.3 Discussion of control performance

In this section, we discuss the control performance by using the transfer function and Bode diagram of the objective model. First, adding Equation (2) to Equation (7), we obtain the following relationship:

\[ \dot{m}V_G + m_0 \dot{y}_G + M \ddot{y} = f_v - (m + m_0 + M)g. \]  

(18)

Applying Equations (7), (9) and (18), the transfer function of the objective model is derived. The following modified variables are newly defined to set the equilibrium points at initial points with zero values:

\[ F_C = f_v - f_{v0}, \quad F_H = f_H - f_{H0}, \quad F_e = f_e - f_{e0}, \]
\[ F_k = f_k - f_{k0}, \quad \ddot{y} = \ddot{y}_0, \quad \dot{y}_0 = \dot{y}_{00}, \quad \dot{y}_G = \dot{y}_{G0}. \]

Applying Laplace transformation to Equations (7), (9) and (18), we obtain the following matrix form:

\[
\begin{pmatrix}
(m + m_0) s^2 + m_0 s^2 - 0 \\
ms^2 + m_0 s^2 - 0 \\
ms^2 + m_0 s^2 - Ms^2
\end{pmatrix}
\begin{pmatrix}
Y_0 \\
Y_0 \\
Y
\end{pmatrix}
= \begin{pmatrix}
(1 + \mu) F_C + (1 - \lambda) G_{eG} \left[ Y - (1 - \lambda) Y_0 - \lambda \dot{y}_G \right] \\
F_C + G_{eG} \ddot{y} + G_{H} \left[ Y - (1 - \lambda) \dot{y}_G - \lambda \ddot{y}_G \right] \\
F_k + G_{eG} \ddot{y} + G_{H} \left[ Y - (1 - \lambda) \dot{y}_G - \lambda \ddot{y}_G \right]
\end{pmatrix},
\]

(19)

where \( sC_i + K_i = G_i(s), \ sC_H + K_H = G_H(s). \)

The output \( s^2 Y(s) \) is represented by both the operational input \( F_d(s) \) and the external force \( F_d(s) \) from Equation (19), as follows:

\[
s^2 Y(s) = \frac{a_2(s) s^2 + a_0(s)}{b_2 s^3 + b_1(s) s^2 + b_0(s)} F_d(s).
\]

(20)

Here, new symbols are defined as follows.

\[
a_2(s) = \left[ m_c - \mu \lambda m + (1 + \mu)(1 - \lambda) m_0 \right] G_{eG}(s)
+ \left[ m_c - \mu m \right] G_{H}(s),
\]
\[
a_0(s) = (1 - \lambda) G_{eG}(s) G_{H}(s),
\]
\[
b_2(s) = b_2 = M \left[ m m_0 + m_c m_0 + m_0 m \right].
\]
also confirmed to be similar to that in those of 13[Hz] and 100[Hz] by virtue of feedback resonant frequencies of 4[Hz] and 35[Hz] transfer to frequency of 35[Hz] is largely reduced, that about the shown in gain diagram with control corresponds to that of the described in Equation (20). It is noticed that the

\[
h_2(s) = \frac{M}{s^2} \left( m + m_0 \right) G_i(s) + \left( \lambda^2 m + m_0 + (1 - \lambda)^2 m \right) G_{i\alpha}(s),
\]

\[
b_2(s) = (1 - \lambda^2) \left( M + m_0 + m \right) G_i(s) G_{i\alpha}(s),
\]

\[
C_z(s) = m m_0 + m_0 m_0 + m m_0,
\]

\[
C_v(s) = (1 - \lambda) \left( 1 - \lambda \right) m_0 - \lambda m \left( G_{i\alpha}(s) - \lambda \right).
\]

Here we represent the Bode gain diagram from the disturbance input \( f \) to the acceleration output \( y \), based on Equation (20). It is noticed that the operational input \( F_2(s) \) includes the properties of the actuator and LPF as shown in Figure 7.

The green curves and blue curves in Figure 14 show the result with the feedback signal of only relative velocity and that without control, respectively. The assumed condition of the Bode gain diagram with control corresponds to that of the acceleration response shown in Figure 11. We can see that the two resonant frequencies of 4[Hz] and 35[Hz] transfer to those of 13[Hz] and 100[Hz] by virtue of feedback control. Although the amplitudes of the past resonant frequency of 35[Hz] is largely reduced, that about the new frequency of 100[Hz] rather increases. The tendency of amplitudes that appeared in Figure 14 is also confirmed to be similar to that in Figure 11.

![Fig. 14 Bode gain diagram-1](image)

**Fig. 14 Bode gain diagram-1**

Figure 15 shows the case with the additional feedback signals of the cart acceleration and the velocity of bearing. The blue curve in Figure 15 is same as that in Figure 14. The assumed condition of the Bode gain diagram with control corresponds to that of the acceleration response shown in Figure 13. We can confirm that the amplitude of the resonant frequency of 100[Hz] becomes smaller and flatter than that shown in Figure 14, and consequently the residual vibration of the cart rapidly converges to zero as shown in Figure 13.

![Fig. 15 Bode gain diagram-2](image)

**Fig. 15 Bode gain diagram-2**

### 5 Conclusion

This paper reported an active controlled caster to reduce the cart crashes and vibrations. The main results are as follows:

1) The active controlled caster was proposed, which was designed under the concept of center of percussion.

2) The dynamical model of the cart with an active caster was derived including the actuator dynamics to study impulsive accelerations and control design.

3) The experimental results had good agreements with the simulation results calculated using the dynamical model.

4) The cart acceleration became about 1/4 and the damping time of the residual vibration became about 1/5 by virtue of feedback control.

5) The control performances were discussed using the Bode gain diagram of the dynamical model.

In this study, the distribution and combination of feedback gains were determined by trials and errors, according to the observation of the actual responses. The systematic determination of gains is left in future works. It is also important for the dynamical model of the cart to be improved to be applicable to not only the vertical motion but also the horizontal motion.

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### References


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